Iterative processes and Navier Stoke. (A Letter)

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Abstract.
We continue with iterative analysis of the Navier Stoke Equations by examining the process by which a solution to any PDE is formed. We then try to apply these ideas to the Navier Stoke Equations.

INTRODUCTION.

1. Thoughts.

Our process uses the following as a basis. As arbitrary PDE's are (or at least can be described as) iterative processes at an infinitesimal level, consider that with each infinitesimal iteration some functional is formed (whether we can actually find it is irrelevant) naturally used in evaluating the constituents of a quantity for use in the evaluation of the solution of the next iteration, and consider that in each such iteration, such a functional exists, expressing the solution at $t, t+\delta$, which is of the form : $F(t) : f(X,Y,Z) \otimes U(X,Y,Z,T) : f \otimes U$. The following observations can then be made of the system involved in the iterative formation of a functional solution to a PDE.

If the quantity $\mathcal{M}$[1] is 'smooth' with respect to all variables, then what can we expect of :

$$\frac{f \otimes U(t+\delta) - f \otimes U(t)}{\delta} \tag{1}$$

For instance only under certain conditions can a limit of a difference divided by δ yield a non-asymptotic form. Logically, it seems clear that $f \otimes U = \int \mathcal{M}_{t+\delta} dt$ [2] is also 'smooth' as for one, it represents a function for which $\mathcal{M}_{t+\delta}$ is the gradient[3]. In the next iteration, this 'function' is used

[1] $\mathcal{M} = (\boldsymbol{u} \cdot \boldsymbol{\nabla})\,\boldsymbol{u} - (-\frac{1}{\rho}\boldsymbol{\nabla} p + \nu \nabla^2 \boldsymbol{u} + f)$

[2] strictly for use as a function to be used for the next iteration

[3] Another argument can be made that, using a series expansion of the functional solution at $(t, t+\delta) : S_n[F(t)]$ requires that : $(S_n[F(t+\delta)] - S_n[F(t)])/\delta$ must be of the form of the expansion of $\mathcal{M}_\delta$ over $(t, t+\delta)$. Thus an expansion of the form : $S_n[F(t)]$, can't be asymptotic for any t, as long as it is linked to $\mathcal{M}_\delta$ in this manner.

to compute $\mathcal{M}_{t+2\delta}$[4] again for the computation of $\int \mathcal{M}_{t+3\delta}$. It is easy to see that the solution to the PDE follows as : $\int \mathcal{M}_t dt|T : (t-\delta, t), \int \mathcal{M}_{t+\delta} dt|T : (t, t+\delta), ..., \int \mathcal{M}_{t+k\delta} dt|T : (t+(k-1)\delta, t+k\delta)$. We are interested in when this series blows up.
It will become clear shortly, however logic dictates that if at any point in the above series, $\int \mathcal{M}_{t+j\delta} dt|T : (t+(j-1)\delta, t+j\delta)$ yields a functional that is asymptotic in nature, thus yielding a value at $T = t + j\delta$ forming part of the domain of such a function, then the solution to the PDE explodes, for the simple reason that the structure associated with the iterative process involved in NS is structured in this manner that once this occurs, every iteration results in a value forming part of some such domain.
It is easy to see that, unless an asymptotic $F(t) : U$ is fed into the constituents of $\mathcal{M}_{t+j\delta}$, the nature and setup of $\mathcal{M}_{t+j\delta}$ is such that it will not convert (in its process) $F(t)$ into an asymptotic function[5]. Thus, unless $\mathcal{M}_{t+j\delta}$ is asymptotic at some iteration (i.e. returns an asymptotic functional at $T = t + j\delta$), $F(t)$ will not be asymptotic! The reasons will become clear in the paragraph after the proceeding observation:

Using these observations, let us use a trace variable K_δ, as a means of following the course of the solution formed by N.S at each interval $(t, t+\delta)$, for input functions that are everywhere 'smooth'.
In just the portion of $\mathcal{M}$, $\mathbf{u}\nabla\mathbf{u}$ replacing δt with K_δ some arbitrarily small constant, the iterations follow the process:

$$(f \otimes UK_\delta \nabla f \otimes UK_\delta)K_\delta$$
$$([f \otimes UK_\delta \nabla f \otimes UK_\delta]K_\delta \nabla [f \otimes UK_\delta \nabla f \otimes UK_\delta]K_\delta)K_\delta$$
$$...$$

Taking the limit of K_δ shows that unless $f \otimes U$ is either asymptotic or the process above is logically unable to convolute such a function into one. The reason is quite deep and can be better seen in the following argument.
At each iteration, since $\int \mathcal{M}_{t+j\delta}$,given that its previous iteration $\mathcal{M}_{t+(j-1)\delta}$ is smooth, returns a functional, the derivative of which is everywhere 'smooth', if $\mathcal{M}_{t+i\delta}$,for some i^{th} iteration, forms part of the domain of an asymptotic function, then so is the functional, the derivative of which (w.r.t t) expresses $\mathcal{M}_{t+j\delta}$ at the iteration in concern, thus , so will $(\mathcal{M}_{t+(j+1)\delta}, \int \mathcal{M}_{t+(j+1)\delta})$ etc.., thereby starting the process leading to an explosive solution. This

[4]as the nature of the structure of $\mathcal{M}$ changes with each iteration, the subscript is used to distinguish one from the other

[5]i.e. the constituents of $\mathcal{M}_{t+j\delta}$ such as $\mathbf{u}\nabla\mathbf{u} : F(t)\nabla F(t)$ does not render an asymptotic function, for everywhere 'smooth' $F(t)$.

expresses clearly the conditions under which N.S explodes. Since in the argument above, j is arbitrary, if $\mathcal{M}_{(0,0+\delta)}$ is everywhere 'smooth', so is the solution.

References

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