

About Superluminal Propagation of an Electromagnetic Wavepacket Inside a Rectangular Waveguide

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Abstract. — We discuss the propagation of an electromagnetic wavepacket inside a rectangular waveguide, of the type employed in recent experiments on superluminal tunneling of electromagnetic signals. By exploiting the analogy between particle and photon tunneling, we consider both evanescent and growing waves inside the narrowed part of the waveguide. The Fourier expansion of such waves shows that the barrier behaves in a nonlocal way. Such a nonlocality is accounted for in an effective way by means of a deformation of the spacetime inside the waveguide. As a consequence, the wavepacket propagates at superluminal speed according to an effective metric tensor, built up in analogy with the Cauchy stress tensor in a deformable medium.

1. Introduction

In the last years, there has been a renewed interest on superluminal processes, due to some new experimental evidences in different sectors of physics. Those include, *e.g.*, the apparent superluminal expansions of galactic objects [1] and the evidence for superluminal motions in electrical and acoustical engineering [2]. However, perhaps the most interesting experimental findings are those concerning the superluminal tunneling of evanescent waves and photons [3–7], first observed at Cologne [3] and Berkeley [5], and then confirmed by a Florence [6] and a Vienna [7] group.

From the theoretical point of view, evanescent waves were predicted to be superluminal [8] on the basis of the analogy between tunneling photons and tunneling particles [9] (which, as is well known, can move with superluminal speed inside the barrier - the so-called Hartmann effect [10]). Some aspects of the superluminal propagation of electromagnetic wavepackets were discussed in references [8–15].

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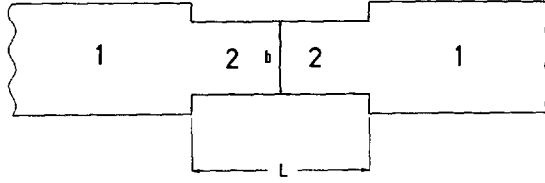


Fig. 1. — Rectangular waveguide with variable section. L = length of the smaller waveguide; b = width of the smaller waveguide; the numbers 1 and 2 refer, respectively, to the regions inside the larger and the smaller waveguide.

In the present paper, we want to discuss in detail the superluminal propagation of an electromagnetic wavepacket inside a rectangular waveguide. The main tools we shall exploit to this aim are: the analogy between particles and waves [9,10,16] and the formalism of deformed Minkowski space [17–19].

The paper is organized as follows. In Section 2 we introduce the basic notions concerning the propagation of the electromagnetic wavepacket inside the waveguide, by exploiting the analogy between particle and photon tunneling. In Section 3 we Fourier-expand the wavepacket inside the narrow part of the waveguide, calculate the partial fluxes and time delays, and show that the barrier behaves in a nonlocal way. Such a nonlocality is described, in Section 4, in terms of a deformation of the Minkowski space-time, which allows us to account for the superluminal propagation of each Fourier component of the wave packet. Section 5 concludes the paper.

2. Helmholtz Equation for an Electromagnetic Wavepacket

Let us consider a hollow rectangular waveguide with variable section (like that used in the Cologne experiment [3]), where the narrow part has length L , height b and thickness a (see Fig. 1). Inside the waveguide, any of the vector quantities \mathbf{f} describing the electromagnetic field ($\mathbf{f} = \mathbf{A}$, \mathbf{E} , or \mathbf{H} , where \mathbf{A} is the vector potential with the subsidiary gauge condition $\text{div } \mathbf{A} = 0$, $\mathbf{E} = -(1/c)\partial\mathbf{A}/\partial t$ is the electric field and $\mathbf{H} = \text{rot } \mathbf{A}$ is the magnetic field) is ruled by the Helmholtz equation

$$\square \mathbf{f} = \nabla^2 \mathbf{f} - \frac{1}{c^2} \frac{\partial^2 \mathbf{f}}{\partial t^2} = 0. \quad (2.1)$$

The general solution of equation (2.1) is given by a wavepacket of the kind

$$\mathbf{f}(\mathbf{r}, t) = \int_{(k>0)} \frac{d^3 k}{k} \chi(\mathbf{k}) a_{\mathbf{k}}(\mathbf{r}) e^{-i c k t} \quad (2.2)$$

where, as usual, $k_0 = \omega/c = \epsilon/\hbar c$, $k = |\mathbf{k}| = k_0$, and

$$\chi(\mathbf{k}) = \sum_{i=1}^2 \chi_i(\mathbf{k}) \mathbf{e}_i(\mathbf{k}) \quad (2.3a)$$

$$\mathbf{e}_i \cdot \mathbf{e}_j = \delta_{ij}; \quad \mathbf{e}_i(\mathbf{k}) \cdot \mathbf{k} = 0, \quad i, j = 1, 2. \quad (2.3b)$$

The vectors \mathbf{e}_j are the (linearly independent) polarization vectors, and $\chi_i(\mathbf{k})$ is the amplitude for the photon to have momentum \mathbf{k} and polarization i , so that $|\chi_i(\mathbf{k})|^2 d^3 k$ is proportional to

the probability that the photon has a momentum between \mathbf{k} and $\mathbf{k} + d\mathbf{k}$ in the polarization state \mathbf{e}_i . Moreover, we have (assuming the z -axis along the waveguide)

$$a_{\mathbf{k}}(\mathbf{r}) = \begin{cases} e^{i\mathbf{k} \cdot \mathbf{r}} + a_R e^{-ik_z z + ik_x x + ik_y y} & \text{region 1} \\ (\alpha e^{-\chi z} + \beta e^{+\chi z}) e^{ik_x x + ik_y y} & \text{region 2} \end{cases} \quad (2.4)$$

i.e. a linear superposition of plane waves in region 1 and a linear superposition of an evanescent and an increasing wave in region 2. On account of the boundary conditions and for transverse electric (TE) waves the following relations hold:

$$k_x = \frac{m\pi}{a}, \quad k_y = \frac{n\pi}{b}; \quad (m, n \text{ integers}) \quad (2.5)$$

$$\left(\frac{m}{2a}\right)^2 + \left(\frac{n}{2b}\right)^2 = \left(\frac{1}{\lambda_c}\right)^2 \quad (2.6)$$

$$\chi = 2\pi \sqrt{\left(\frac{1}{\lambda}\right)^2 - \left(\frac{1}{\lambda_c}\right)^2} = \sqrt{k_x^2 + k_y^2 - \frac{\omega^2}{c^2}} = \left(\frac{1}{\hbar c}\right) \sqrt{\left(\frac{2\pi\hbar c}{\lambda_c}\right)^2 - \epsilon^2} \quad (2.7)$$

where λ_c is the cutoff wavelength.

Let us explicitly notice that the need of taking into account *both* evanescent *and* growing waves in region 2 is demanded by the analogy between photon and particle tunneling [16]. By the same analogy, one gets the last expression of χ in equation (2.7); moreover, it can be further exploited in the calculation of the stationary flux. Indeed, we have, for particles of mass μ and wavefunction $\psi(z)$:

$$j = \text{Re} \left[\left(\frac{i\hbar}{2\mu} \right) \psi \frac{\partial \psi^*}{\partial z} \right]. \quad (2.8)$$

On the other side, the z -component of the Poynting vector is given by

$$S_z = \frac{c}{4\pi} \text{Re}[\mathbf{E}^* \times \mathbf{H}]_z = \frac{c}{4\pi} \text{Re} \left[E_x^* \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right) - E_y^* \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right) \right]. \quad (2.9)$$

For monochromatic TE waves, we have

$$\begin{cases} E_x = -E_0 \frac{k_y}{k_x} \cos k_x x \cdot \sin k_y y e^{i\omega t} \psi(z) \\ E_y = E_0 \sin k_x x \cdot \cos k_y y e^{i\omega t} \psi(z) \\ E_z = 0 \end{cases} \quad (2.10)$$

where

$$\psi(z) = \begin{cases} e^{ik_z z} + a_R e^{-ik_z z} & \text{region 1} \\ \alpha e^{-\chi z} + \beta e^{+\chi z} & \text{region 2} \end{cases} \quad (2.11)$$

Therefore, the vector potential is

$$\begin{cases} A_x = a_x(x, y) \psi(z) e^{-i\omega t} \\ A_y = a_y(x, y) \psi(z) e^{-i\omega t} \\ A_z = 0 \end{cases} \quad (2.12)$$

with

$$\begin{cases} a_x(x, y) = c_x e^{ik_x x + ik_y y} \\ a_y(x, y) = c_y e^{ik_x x + ik_y y} \end{cases} \quad (2.13)$$

Replacing equations (2.12, 2.13) in (2.9), we get the following expression of the Poynting vector component:

$$S_z = \text{Re}[|a_x|^2 + |a_y|^2][\psi(z)(\frac{-i\omega}{4\pi})\frac{\partial\psi^*}{\partial z}]. \quad (2.14)$$

Therefore, the flux S_z for photons is obtained from the flux for particles by simply replacing in equation (2.8) $(-i\hbar/2\mu)$ by $[(a_x)^2 + (a_y)^2](-i\omega/4\pi)$.

3. Fourier Analysis of the Wavepacket Propagation

We want now to discuss in detail the propagation of the electromagnetic wavepacket inside the narrow part of the waveguide (region 2). Let us first consider the evanescent wave in region 2, given by equation (2.4b), and expand it in a Fourier series:

$$e^{-\chi z} = \sum_{n=-\infty}^{\infty} c_n e^{in2\pi z/L} \quad (3.1)$$

where (for $\chi L \gg 1$)

$$c_n = \frac{(1/L)}{\chi + in\frac{2\pi}{L}}. \quad (3.2)$$

We can calculate the stationary flux of the monochromatic wavepacket by exploiting the analogy with particle tunneling as shown in the previous section. So the flux for the evanescent wave (3.1) is given by (2.8). After lengthy but straightforward calculations, we get

$$j = \frac{\pi\hbar}{\mu L} \left(\frac{1}{L}\right)^2 \sum_{m,n} m \frac{\{[\chi^2 + mn(\frac{2\pi}{L})^2] \cos[(n-m)\frac{2\pi}{L}z] + (m-n)^2 \frac{2\pi}{L} \chi \sin[(m-n)\frac{2\pi}{L}z]\}}{[\chi^2 + n^2(\frac{2\pi}{L})^2][\chi^2 + m^2(\frac{2\pi}{L})^2]} \quad (3.3)$$

Notice that

$$\begin{aligned} \sum_{m,n} &= \sum_{m=n} + \sum_{m>n} + \sum_{m<n}; \\ \sum_{m>n} &= - \sum_{m<n} \rightarrow \sum_{m>n} + \sum_{m<n} = 0. \end{aligned}$$

Therefore

$$j = \frac{\pi\hbar}{\mu L} \left(\frac{1}{L}\right)^2 \sum_{n=-\infty}^{\infty} \frac{n}{\chi^2 + n^2(\frac{2\pi}{L})^2} = 0 \quad (3.4)$$

as expected. So, we can analyze only the partial fluxes j_n , given by

$$j_n = \frac{\pi\hbar}{\mu L} \left(\frac{1}{L}\right)^2 \frac{n}{\chi^2 + n^2(\frac{2\pi}{L})^2} \quad (3.5)$$

where $n > 0$ corresponds to incoming waves, and $n < 0$ to returning waves.

For quasi monochromatic wave packets, the n -th term of the sum in (3.1) must be replaced by

$$c_n e^{(in2\pi z/L) - (i\epsilon t/\hbar)} \quad (3.6)$$

Then, it is easy to get, by a standard procedure (in the stationary phase approximation) [9], the partial time delays:

$$\Delta\tau_n = \hbar \frac{\partial \arg c_n}{\partial \epsilon} = \hbar \frac{\partial \arctg(\frac{n2\pi}{L\chi})}{\partial \epsilon} = \hbar \frac{1}{1 + (\frac{2\pi n}{L\chi})^2} \frac{2\pi n}{L\chi^2} \frac{\partial \chi}{\partial \epsilon} \quad (3.7)$$

which, as is well known, do represent the delays in the arrival of the wave maxima. Since

$$\frac{\partial \chi}{\partial \epsilon} = -\frac{k_z}{c\chi}, \quad (3.8)$$

we find finally

$$\Delta\tau_n = -\hbar \frac{2\pi n k_z}{L c \chi [\chi^2 + (\frac{2\pi n}{L})^2]}. \quad (3.9)$$

We have therefore that (like in the particle case)

- i) $n < 0$ corresponds to a *delay* of the wave;
- ii) $n > 0$ corresponds to an *advance* of the wave.

For the growing wave $e^{+\chi z}$ with Fourier series

$$e^{+\chi z} = \sum_{n=-\infty}^{\infty} d_n e^{in \frac{2\pi}{L} z}, \quad (3.10)$$

we get (since $\chi \rightarrow -\chi$) analogous but *interchanged* results.

On the basis of the above results, we can therefore conclude that:

- a) The evanescent wave (3.1) contains two *kinds of waves*: *incoming* (moving toward the positive direction of the z -axis) and *returned* (*i.e.* reflected by the barrier - region 2 - as a whole);
- b) In the growing wave (3.10), there are still two kinds of waves, but of origin *different* from the evanescent case, namely: waves *reflected by the "second barrier well"* (junction $2 \rightarrow 1$) and *secondly returned by the barrier* - region 2 - as a whole (*i.e.* still moving in the positive z -axis direction, as the initially incoming ones).

Thus, the linear combination (2.11b) of the evanescent and the growing wave contains *all the above four kinds of current waves*. They give rise to *nonzero fluxes* j_+, j_- (where j_+, j_- are the positive and the negative component of the total flux-according to reference [9] - associated with motion along the positive and the negative z -direction, respectively).

By the way, in all the sums the term $n = 0$ corresponds to *zero flux*. This may be interpreted as representing *the time-integral probability of particle (photon) dwelling*, due to the accumulating (to and from) current waves.

4. Nonlocality of Propagation and Deformed Minkowski Space

Let us notice that the behaviour of the barrier (region 2) is *nonlocal*. This is due to the fact that (both in the case of the evanescent and the growing wave) *the waves are reflected by the barrier as a whole* (see points a), b) of the previous section).

Such nonlocal effects in the propagation of the wavepacket allow us to clarify a basic point of our approach. Indeed, what is the physical meaning of the *real momenta* which appear in the Fourier transforms of the evanescent and the growing wave?

This point can be understood by noting that nonlocal effects inside a narrowed waveguide can be taken into account in an effective way by means of a deformed Minkowski spacetime [17, 18]. Namely, according to reference [17], an evanescent mode in the usual Minkowski space is described as a non-evanescent one, with a real wavevector, propagating at a superluminal speed in a deformed Minkowski space endowed with metric

$$\eta = \text{diag}(b_0^2, -b_1^2, -b_2^2, -b_3^2) \quad (4.1)$$

where the metric parameters b_μ^2 are functions of the energy of the single photon:

$$b_\mu^2 = b_\mu^2(E), \quad \mu = 0, 1, 2, 3. \quad (4.2)$$

In our case, E corresponds to the energy of the Fourier component (see below).

Wave propagation in such a spacetime is described by the deformed Helmholtz equation [17,18]

$$\begin{cases} \tilde{\square} f = 0 \\ \tilde{\square} = (b_0^2/c^2)\partial t^2 - b_1^2\partial_x^2 - b_2^2\partial_y^2 - b_3^2\partial_z^2 \end{cases} \quad (4.3)$$

An analysis of the energy dependence of the metric parameters (performed by using the data of the Cologne experiment reported in Ref. [3]) shows that the metric η reduces to the usual Minkowskian one, $g = \text{diag}(1, -1, -1, -1)$, for energies $E > E_0 \sim 4, 5 - 5 \mu\text{eV}$ [18]. So, in our case, the metric is fully Minkowskian in region 1, and becomes deformed in region 2 (where $E < E_0$) [17]. In particular, any Fourier component of the wavepacket satisfies the deformed Helmholtz equation (4.3) in region 2. Without loss of generality (since all quantities of interest to us depend on the ratio b_0^2/b_3^2), we can assume that the deformed metric is isochronous with the usual Minkowskian one, *i.e.* $b_0^2 = 1$. So, the deformed wavevector along z for the n -th Fourier component is given by [17]

$$(\tilde{k}_z^{(n)})^2 = \left(\frac{2\pi}{c}\right)^2 \left[\left(\frac{\nu}{b_3^{(n)}}\right)^2 - \nu_c^2 \right] \quad (4.4)$$

where ν_c is the cutoff frequency. Since $\tilde{k}_z^{(n)} = 2\pi n/L$, we get the explicit expression of the parameter $b_3^{(n)}$.

$$b_3^{(n)} = \frac{\nu}{\sqrt{\nu_c^2 + (nc/L)^2}} = \frac{L\nu/n}{\sqrt{c^2 + (L\nu_c/n)^2}}. \quad (4.5)$$

The speed u_n of the n -th Fourier component therefore reads

$$u_n = \frac{c}{b_3^{(n)}} = c \frac{\sqrt{c^2 + (L\nu_c/n)^2}}{L\nu/n} \quad (4.6)$$

Since

$$\frac{\sqrt{c^2 + (L\nu_c/n)^2}}{L\nu/n} > 1 \quad (4.7)$$

for

$$\nu < \nu_c \quad (4.8)$$

(and $n > 0$), we always have

$$u_n > c \quad (4.9)$$

i.e. all the propagating waves in the Fourier expansions (3.1), (3.10) are superluminal. Let us recall that the “superluminality condition” (4.8) [17] is in fact satisfied by the parameter values of the Cologne experiment (with $(\nu_c/\nu)^2 = 1.2$).

Notice that each Fourier component propagates in a different deformed Minkowski spacetime. This is clearly related to the energy (and momentum) dependence of the parameters of the deformed metric (4.1). If $\eta_{\mu\nu}^{(n)}$ is the deformed metric “seen” by the n -th Fourier component of the evanescent wave, we can build up an *effective metric tensor* $\bar{\eta}_{\mu\nu}$ for the evanescent wave as follows:

$$\bar{\eta}_{\mu\nu}(c_n) = \frac{\sum_n |c_n|^2 \eta_{\mu\nu}^{(n)}}{\sum_n |c_n|^2} \quad (4.10a)$$

where the c_n 's are the coefficients of the Fourier expansion (3.1).

The analogous definition for the growing wave is

$$\bar{\eta}_{\mu\nu}(d_n) = \frac{\sum_n |d_n|^2 \eta'_{\mu\nu}(n)}{\sum_n |d_n|^2} \quad (4.10b)$$

where the d'_n 's are the coefficients of the expansion (3.10). Notice that, in general, the n -th Fourier component of the evanescent and the growing wave will see a *different* metric deformation (i.e. $\eta_{\mu\nu}^{(n)} \neq \eta'_{\mu\nu}(n)$) due to the connection among the wavevector and the metric parameters (see Eq. (4.4)), and the dependence of the latter ones on the energy (Eq. (4.2)).

The total effect of metric deformation is therefore represented by a linear combination of the tensors (4.10) for both the evanescent and the growing wave:

$$\bar{\eta}_{\mu\nu} = |\alpha|^2 \bar{\eta}_{\mu\nu}(c_n) + |\beta|^2 \bar{\eta}_{\mu\nu}(d_n) = |\alpha|^2 \frac{\sum_n |c_n|^2 \eta_{\mu\nu}^{(n)}}{\sum_n |c_n|^2} + |\beta|^2 \frac{\sum_n |d_n|^2 \eta'_{\mu\nu}(n)}{\sum_n |d_n|^2} \quad (4.11)$$

where α, β are the coefficients in equation (2.11b).

Clearly, in region 1 (where all Fourier waves do propagate in a Minkowskian spacetime), one recovers, from definitions (4.10, 4.11), the usual metric g .

In fact, in region 1 we have $E > E_0$, so that $b_\mu^2(E) = 1$, $\mu = 0, 1, 2, 3$ (see Refs. [17, 18]), and therefore $\eta_{\mu\nu}^{(n)} = g_{\mu\nu} \forall n$. So, all definitions (4.10, 4.11) reduce to the Minkowskian metric.

Notice that the tensors (4.10) are analogous to the Cauchy stress tensor. Indeed, let us consider, in orthogonal Cartesian coordinates, an infinitesimal tetrahedron with edges parallel to the coordinate axes and the oblique face S opposite to the vertex 0 , origin of the Cartesian frame.

If the tetrahedron is a part of a continuous body, the stress vector across S in the point 0 is given by [20]:

$$(\phi_a)_0 = \frac{\sum_i \phi_i a^i}{\sum_i |a^i|^2} \quad (4.12)$$

where \mathbf{a} is a vector normal to S and $(\phi_i)_0$ ($i = 1, 2, 3$) is the stress vector on the face of the tetrahedron orthogonal to the i -th axis. The nine components of the three vectors $(\phi_i)_0$ do just constitute the rank-two, symmetric Cauchy tensor.

The tensor $\bar{\eta}$ can be therefore regarded as the average tensor representing the space-time deformation in the region 2 of the waveguide (corresponding to the energy $E < E_0 \sim 4.5 \mu\text{eV}$) globally "seen" by the electromagnetic Fourier components for the wavepacket (2.11b). So, we can name it *average tensor of the electromagnetic space-time deformation*, $\bar{\eta}_{\text{e.m.}}$.

It is worth stressing that our approach to electromagnetic faster-than-light propagation in waveguides is similar, in some respects, to that where superluminal propagation (e.g. of light between parallel mirrors) is connected to vacuum effects [21]. In such a case, the influence of the (structured) vacuum is described in an effective way in terms of a refractive index (as pioneered by Sommerfeld). Something analogous does happen in General Relativity, too: the deflection of light rays in a gravitational field can be considered as a propagation in an Euclidean space, filled with a medium endowed with an effective refractive index [22, 23]. In some cases, such

a propagation, due to the influence of the gravitational vacuum, turns out to be superluminal (the refractive index is less than one) [24]. Notice that our approach can be therefore regarded as *dual* to the general relativistic one, in which the spacetime curvature for electromagnetic signals is replaced by a refractive index. Indeed, in our formalism, *the vacuum or nonlocal effects which affect propagation in the waveguide are directly described in terms of a spacetime deformation* (and the role of the refractive index is played by the deformation tensor).

Moreover, it is easily seen that definitions (4.10, 4.11) can also be applied to wavepackets ruled by interactions different from the e.m. one [17,19]. Namely, we can state in full generality that the Minkowski space is *always* subjected to a stress, whenever crossed by a wavepacket. Such a stress produces a deformation of the spacetime, which may be described by the tensor $\bar{\eta} = g$ (*ineffectual deformation*) or by a tensor $\bar{\eta} \neq g$ (*effectual deformation*). The two cases $\bar{\eta} = g$, $\bar{\eta} \neq g$ are obviously determined by the interaction ruling the wavepacket propagation and by the energy of the wavepacket components (see Eq. (4.2)).

The superluminal propagation, which is an unescapable consequence of the tensor (4.10) in case of effectual deformation, does no longer imply a violation of causality in the deformed Minkowski space ⁽¹⁾. This is in fact related to the invariance of the tensor (4.10) under the generalized Lorentz transformations valid in the deformed Minkowski space (see Refs. [16, 17, 19]).

5. Conclusions

In this paper, we have discussed the propagation of an electromagnetic wavepacket inside a hollow waveguide with variable section. On the basis of the analogy between particle tunneling and photon tunneling, we have shown that a complete analysis of the process requires considering *both* an evanescent *and* a growing wave inside the waveguide. We have Fourier-analyzed such waves, and stressed that both of them contain returned waves, which are due to the action of the barrier (narrowed part of the waveguide) as a whole. Such a behaviour of the barrier is therefore nonlocal. As a consequence, we can reinterpretate the propagation of each Fourier component inside the barrier as a wave propagation in a deformed Minkowski spacetime. This implies a *real* wavevector for each Fourier wave, and *superluminal* propagation of each Fourier component. So our approach leads in a straightforward way to explaining the superluminal tunneling of the wavepacket as its propagation in a region with deformed metric. Such a spacetime deformation can be described, in an effective way, by an average symmetric tensor $\bar{\eta}_{\mu\nu}$, built up from the deformed metrics “seen” by each Fourier component of the wavepacket, in analogy with the well-known definition of the Cauchy stress tensor for a deformable medium.

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⁽¹⁾ This is related to the fact that, inside a deformed Minkowski space, the maximal causal speed is, in general, greater than c , depending on the metric parameters. See references [18,19], and references therein.

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- [21] Barton G. and Scharnhorst K., *J. Phys. A* **26** (1993) 2037 and references therein.
- [22] Such a result can be traced back to T. Levi-Civita, *The Absolute Differential Calculus* (Blackie and Son, 1954) p. 403.
- [23] See *e.g.* Birrell N.D. and Davies P.C.W., *Quantum Fields in Curved Space* (Cambridge, 1982); Fulling S.A., *Aspects of Quantum Field Theory in Curved Space-Time* (Cambridge Univ. Press, Cambridge, 1989) and references therein.
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