

Energy-aware 3-level coding and control co-design for sensor network systems

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Abstract—This paper addresses issues on control and coding co-design in the context of low-energy sensors. We particularly focus on issues of low energy consumption (energy-aware). To this aim, we propose to use a coding strategy with the ability to quantify and to differentiate stand-still signal events from changes in the source (level crossing detector). The stand-still signal event is then modulated with a low energy carrier, whereas the changes of levels will be modulated with enough energy. Coding is then effectuated by defining a 3-valued alphabet. The paper studies the closed-loop properties of such arrangement. In particular, we derive conditions required so that this coding algorithm preserves closed loop stability.

Index Terms—Control of sensor networks, energy-aware coding.

I. INTRODUCTION

WIRELESS low-cost sensor networks are an expanded technology in many new and varied areas such as: traffic monitoring and control (urban, highways), undersea monitoring/exploration, environment sensing (forest, farms, etc.), building services, large instruments with distributed sensing and actuators (Tokomak, telescopes), etc.

Sensors will be packaged together with communication protocols, RF electronics, and energy management systems. Therefore, the development of such integrated sensors will be driven by constraints like: low cost, ease of replacement, low energy consumption, and efficient communication links. In turn, these constraints bring new problems to be considered in the exploitation of this information. For instance, low cost will induce sensors with low resolution (binary sensors, at the extreme), low consumption will impose issues on efficient sensor energy management (sleep and wake-up modes), ease of replacement will imply the system ability to keep safe operation in a failure of one or several sensors, and finally communication links and protocols should be designed to account for energy savings, information loss, and varying fading characteristics.

To some extent, the coding structure proposed here can be interpreted as a particular class of quantizer. Some works in relation to study of coarse quantizers and coding have been previously reported in [1], [2], [3], [4], [5], [6], [7], [8], [9], and [10], [11] among others. The use of 3-valued code, in connection with a delta-modulation coding structure and a variable length-block encoding scheme, was proposed in [12]. In this work, the authors have proposed to use entropy coding to take advantage of the probability distribution of the

events, and hence to improve compression rates. However, no claim for energy saving was done.

To the authors knowledge, this work is one of the first intents to study the coding design in the context of NCS in connection to characteristics of low-energy sensors. In this paper, we particularly focus on issues on low energy consumption (energy-aware). To this aim, we propose to use a coding strategy with the ability to quantify and to differentiate stand-still signal events from changes in the source (level crossing detector). The stand-still signal event is then modulated with a low energy carrier (sleep mode), whereas the changes of levels will be modulated with enough energy (wake-up mode). Coding can then be effectuated by defining a 3-valued alphabet: 0 for the case where the source signal information is contained in the time interval between level crossing(sleep mode) and, ± 1 in order to indicate the direction of the level crossing when it occurs (wake-up mode).

The overall coding strategy studied here is composed of two main blocks: (i) a 3-valued encoder including a model-based predictor (MBP) similar to the one proposed in [12], and (ii) an amplitude modulator used to carry the code produced by the encoder. The paper aims at studying the closed-loop properties of such arrangement. In particular we derive conditions required by this coding algorithm in order to preserve closed loop stability.

A. Definitions

- r_k : reference signal,
- x_k : system output,
- $\hat{x}_k$: estimated (reconstructed) output,
- $\tilde{x}_k$: true estimated error, $\tilde{x}_k = x_k - \hat{x}_k$,
- φ_{LD} : level detector,
- φ_{LD}^{-1} : inverse of level detector,
- $\hat{\tilde{x}}_k$: approximated estimated error, obtained after reconstruction, i.e. $\hat{\tilde{x}}_k = \{\varphi_{LD}^{-1} \circ \varphi_{LD}\}(\tilde{x}_k)$, with $\varphi_{LD}^{-1} \circ \varphi_{LD} \neq 1$.
- Δ : step interval used to detect level and reconstruct $\hat{\tilde{x}}_k$,
- δ_k : 3-level valued integer signal: $\{-1, 0, 1\}$. Signal to be modulated,
- $\hat{\delta}_k$: demodulated signal at the received information,
- u_k : control input.

B. Assumptions

- The transmitted information is amplitude modulated

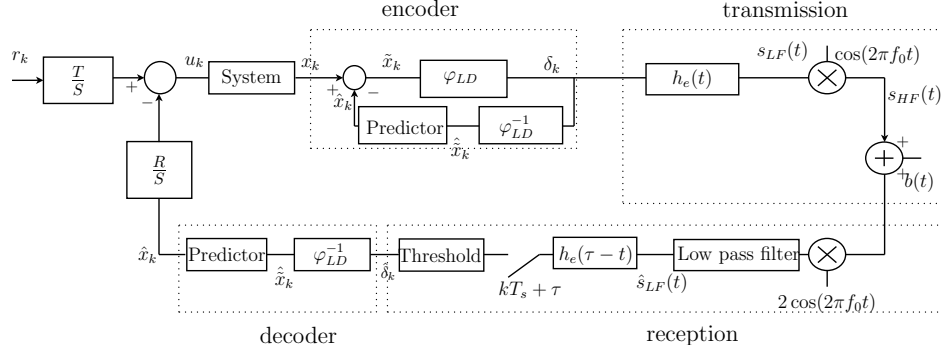


Fig. 1. Block diagram of the complete estimation/control process including the coding and the modulation steps.

- Only encoder-to-decoder information transmission is allowed,
- Reliable noiseless transmission channel is considered,
- local clocks at the encoder/decoder are assumed to be synchronized.

II. PROBLEM SET UP

We consider the following SISO discrete-time linear system (possible unstable), of the form,

$$x_k = \frac{B(q^{-1})}{A(q^{-1})} u_k \quad (1)$$

together with an RST controller,

$$u_k = \frac{R(q^{-1})}{S(q^{-1})} \left\{ \frac{\gamma}{T(q^{-1})} r_k - \hat{x}_k \right\} \quad (2)$$

where r_k is the reference, $\hat{x}_k$ is the estimated of the system output x_k , and $R(q^{-1})$, $S(q^{-1})$, $T(q^{-1})$ are the control polynomials in the delay operator q^{-1} . They also satisfy:

$$T = RB, \quad SA + RB = A_{cl}, \quad \gamma \triangleq A_{cl}(1)$$

with A_{cl} being the closed-loop polynomial, and γ the static gain needed to reach unitary zero-frequency gain. For simplicity, we will omit the use of the argument (q^{-1}) when needed.

The coding process consists in several steps:

- 1) Encoding the system output x_k . This process yields the 3-valued signal δ_k ,
- 2) Modulation of the encoded signal. δ_k is transmitted by using some particular type of modulation, i.e. amplitude shift keying modulation (ASK). Note that when signal is in stand still mode (the produced code is $\delta_k = 0$) then very limited power is needed,
- 3) Demodulation of the transmitted signal $\hat{\delta}_k$, and
- 4) Decodification of the received information to produce the estimated $\hat{x}_k$.

The complete sequence can be seen as a full estimation process. The different components are shown in Figure 1.

When $\hat{x}_k \equiv x_k$, the above controller gives the following closed-loop nominal relation,

$$x_k = \frac{\gamma}{A_{cl}(q^{-1})} r_k$$

else ($\hat{x}_k \neq x_k$), we have,

$$x_k = \frac{\gamma}{A_{cl}(q^{-1})} r_k + W(q^{-1}) \tilde{x}_k$$

where $\tilde{x}_k = x_k - \hat{x}_k$ is the estimation error, and $W = BR/A_{cl}$. As A_{cl} defines a stable polynomial, the output x_k is kept bounded as long as $\tilde{x}_k$ is bounded as well.

The problem is then to design the combined coding/modulation process that defines the output $\hat{x}_k$ from the input x_k preserving closed-loop properties. This process, which will be described next, can be split into two steps: *coding and modulation*. We first present the coding algorithm and study its stability assuming that the transmitted signal process is ideal, then we present in a subsequent section the influence of the amplitude modulator in terms of stability.

III. CODING PROCESS

We assume first that $\hat{\delta}_k = \delta_k$, the effect of the modulation process is first neglected. The coding (encoding/decoding) process is shown in Figure 1. The encoder (respectively the inverse decoder) operation is composed of: a non uniform sampler encoder including a level detector (LD), associated to the map φ_{LD} , together with a model-based predictor (MBP).

A. The Level Detector

The operation principle of the level detector is shown in Figure 2. The map ($\varphi_{LD} : \tilde{x}_k \mapsto \delta_k$), takes the error signal and codes the output signal into a 3-valued $\delta_k \in \{-1, 0, 1\}$. That is:

$$\delta_k = \begin{cases} 1 & \text{if one level is crossed upwards,} \\ 0 & \text{if } \tilde{x}_k \text{ stays at the actual level,} \\ -1 & \text{if one level is crossed downwards.} \end{cases}$$

Equations behind this are:

$$l_k = \left\lfloor \frac{\tilde{x}_k}{\Delta} - \frac{1}{2} \right\rfloor$$

$$\delta_k = f(\tilde{x}_k) = \begin{cases} 0 & \text{if } l_k = l_{k-1}, \\ \text{sign}(l_k - l_{k-1}) & \text{else,} \end{cases}$$

with Δ the level threshold and $\lfloor \cdot \rfloor$ the floor operator which rounds to the smaller integer.

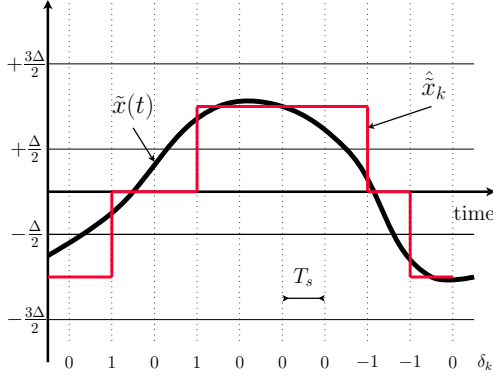


Fig. 2. Illustration of the level detector working operation principle.

B. The model-based predictor (MBP)

The role of the MBP is to recover the encoded signal x_k from the 3-valued binary signal δ_k . It is composed of:

- *The inverse of the level detector:* $\varphi_{LD}^{-1} : \delta_k \mapsto \hat{x}_k$, whose equation is:

$$\hat{x}_k = \hat{x}_{k-1} + \Delta \cdot \delta_k$$

- *The predictor:* it is a model-based predictor. As its name indicates, it uses the target closed-loop model as a basis for its design. This structure is inspired by our previous works in [9], [10], and also in [7]. The predictor is a dynamic linear discrete-time operator that maps the output of the inverse level detector, to the signal prediction $\hat{x}_k$. Its structure depends upon the particular control used (state feedback or output feedback). For instance, for the RST-control discussed here, it has the following form:

$$\hat{x}_k = W \left[\frac{\gamma}{T} r_k + \hat{x}_k \right], \quad W \triangleq \frac{BR}{A_{cl}}, \quad (3)$$

which results in the following error equation:

$$\tilde{x}_k = W \left[\tilde{x}_k - \hat{x}_k \right]. \quad (4)$$

IV. STABILITY PROPERTIES UNDER IDEAL TRANSMISSION

Following the assumptions made in this section (lossless transmission channel, with no transmission delay nor noise), we then have that $\delta_k = \hat{\delta}_k$. In this case, we do not need to differentiate the MBP at the encoder from the MBP at the decoder otherwise it must be, and hence error equation can be described by real variables only.

A. Error system

Introducing:

- $e_k = x_k - \frac{\gamma}{A_{cl}} r_k$: the tracking error,
- $\tilde{x}_k = x_k - \hat{x}_k$: the prediction error, and
- $\varepsilon_k = \tilde{x}_k - \hat{x}_k$: the LD error.

we have the closed-loop error system:

$$e_k = W(q^{-1}) \tilde{x}_k \quad (5)$$

$$\tilde{x}_k = W(q^{-1}) \varepsilon_k \quad (6)$$

with $W = BR/A_{cl}$ being the stable operator defined previously. Note that $\varepsilon_k = \varepsilon_k(\tilde{x}_k)$, and thereby the above error equation can be seen as two systems in cascade, i.e. the output of the autonomous system (6) is the input of the stable system (5). For stability purposes it is thus sufficient to demonstrate the stability properties of the sub-system (6).

Note that ε_k writes as:

$$\varepsilon_k = \tilde{x}_k - \hat{x}_k = \tilde{x}_k - \varphi_{LD} \circ \varphi_{LD}^{-1} \{ \tilde{x}_k \} = \tilde{x}_k - \tilde{\varphi}_{LD} \{ \tilde{x}_k \}$$

where $\tilde{\varphi}_{LD} \triangleq \varphi_{LD} \circ \varphi_{LD}^{-1} : \tilde{x}_k \mapsto \hat{x}_k$. Note that this map is dynamic and defined by the following relation:

$$\hat{x}_k = \hat{x}_{k-1} + \Delta \cdot \delta_k \quad (7)$$

with $\delta_k = f(\tilde{x}_k)$ as defined before. Ideally, ie without coding, the map $\tilde{\varphi}_{LD}$ be a linear map with unitary gain. This ideal goal is hampered by several factors, among which the unknown initial conditions of $\tilde{x}_0$, and, more important, by a bad choice of T_s , and Δ . In particular, large sampling times T_s , and too small quantum Δ may result in signal variation of more than one level, which may lead to unrecovered bias in the estimated, leading to potential instabilities for unstable open-loop systems.

The following analysis gives sufficient stability conditions, and it also details the type of stability that can be reached with this coding scheme. For simplicity reasons, the analysis is presented using a simple linear system with a scalar gain which captures the essential stability characteristics of such type of scheme, namely:

- condition needed for stabilization given as a function of the maximum unstable open-loop eigenvalues, and
- attraction domain specified as a function of the granularity of the level detector (Δ)

The analysis first presents the case of the ideal transmission when $\hat{\delta}_k = \delta_k$, then, in subsequent sections, the impact of the modulation latency is assessed.

B. Stability properties: ideal transmission $\hat{\delta}_k = \delta_k$

Consider the stabilization problem ($r = 0$) of the following simple unstable system $\frac{B(q^{-1})}{A(q^{-1})} = \frac{bq^{-1}}{1-aq^{-1}}$, with $2 > |a| > 1$, and the static feedback control law $u = kx_k$. Let $1 > a_c > 0$ be the desired closed loop poles, the required gain to reach such closed-loop specification is $k = -(a - a_c)/b$. This particular choice leads to the error equations (5)-(6) with $W(q^{-1}) = \frac{(a-a_c)q^{-1}}{1-a_cq^{-1}}$. Due to the cascade structure of such error equation arrangement, stability only relies on the stability of the equation (5) which captures most of the difficulties. To this aim we will concentrate on the following set of equations,

$$\tilde{x}_{k+1} = a_c \tilde{x}_k + (a - a_c) \varepsilon_k, \quad \varepsilon_k = \tilde{x}_k - \hat{x}_k \quad (8)$$

$$\hat{x}_k = \hat{x}_{k-1} + \Delta \text{sign}(l_k - l_{k-1}), \quad l_k = \left\lfloor \frac{\tilde{x}_k}{\Delta} - \frac{1}{2} \right\rfloor \quad (9)$$

The stability analysis mainly depends on the following two properties:

- **Rate level condition.** Defines conditions on a , and a domain B_{ρ_1} for $\tilde{x}_k$ that ensure that no more than one level change can be effectuated, i.e. $|l_k - l_{k-1}| \leq 1$,
- **Invariance condition.** Defines condition under which the domain B_{ρ_1} is indeed an invariant; solutions $\tilde{x}_k$ starting in B_{ρ_1} do not leave this domain.

Lemma 1: Rate level condition. Consider unstable systems limited by the relation $a < 2 + a_c < 3$, and let define the compact set, B_{ρ_1} , as:

$$B_{\rho_1} = \{\tilde{x}_k : |\tilde{x}_k| < \rho_1\}, \quad \rho_1 = \frac{(1 - \frac{(a-a_c)}{2})}{1 - a_c} \Delta$$

with $\rho_1 > 0$. Then for all $|\tilde{x}_k| \in B_{\rho_1}$ the following holds, $\forall k \in \mathbb{Z}^+$:

- i) $|\tilde{x}_k - \tilde{x}_{k-1}| \leq \Delta$, furthermore, i) implies the following two equivalent inequalities:
- ii) $|l_k - l_{k-1}| \leq 1$
- iii) $|\varepsilon_k| \leq \Delta/2$

Proof: . See [12] ■

The Lemma establishes conditions on $|\tilde{x}_k|$, $\forall k \in \mathbb{Z}^+$ such that the rate change in the level detector be at most one. For consistency reason, it is implicitly assumed that the encoder/decoder internal states are suitably initialized. That is, $\hat{x}_0$, and l_0 are such that: $\varepsilon_0 < \Delta/2$, and $\hat{x}_0 = \Delta l_0$ at $k = 0$.

Lemma 2: Invariance condition. Assume that $\hat{x}_0$, and l_0 are such that: $\varepsilon_0 < \Delta/2$, and $\hat{x}_0 = \Delta l_0$ at $k = 0$, and that $\tilde{x}_0 \in B_{\rho_1}$, then if,

$$\rho_0 = \Delta \frac{(a - a_c)}{2(1 - a_c)} < \Delta \frac{(1 - \frac{(a-a_c)}{2})}{1 - a_c} = \rho_1$$

then all solutions of $\tilde{x}_k \in B_{\rho_1}$, for all $k = 0, 1, \dots$

Proof: . See [12] ■

Working out details of the above inequality, it can be shown that this equality holds if $a - a_c < 1$, for all $a_c \in (0, 1)$. Note that this is a stronger condition than the one in Lemma 1 as it is derived from a more conservative (Lyapunov) analysis.

The following theorem used the previous two lemmas to derive the complete stability result.

Theorem 1: Assume that the coding algorithm is initialized such that $\hat{x}_0$, and l_0 are such that: $\varepsilon_0 < \Delta/2$, and $\hat{x}_0 = \Delta l_0$. Consider system satisfying $a - a_c < 1$, with initial condition in the set $\tilde{x}_0 \in B_{\rho_1}$. Then:

- $\tilde{x}_k \in B_{\rho_1}$, $\forall k \in \mathbb{Z}^+$,
- $\exists k_0 : |\tilde{x}_k| \leq \rho_0$, $\forall k \geq k_0$, and
- $\lim_{k \rightarrow \infty} d(x_k, \mathcal{B}_\beta) = 0$.

where $d(x_k, \mathcal{B}_\beta)$ is the minimum Euclidean distance from x_k to any point within the ball

$$\mathcal{B}_\beta := \{x \in \mathbb{R} : \|x\| < \beta\},$$

and β is a constant that depends on ρ_0 , and on the infinite norm of $W(q^{-1})$.

Proof: The first two statements follow from the previous analysis, the last statement result from equation (5), i.e;

$|x_k| \leq \|W\| \cdot |\tilde{x}_k|$. Details for the derivation of this property are similar to the ones used in [9], and [10]. ■

V. STABILITY PROPERTIES WITH WIRELESS TRANSMISSION

In this section we study the stability properties for the case of wireless transmission. We first present the wireless transmission method (amplitude modulation) and investigate the effects of this transmission (latency), and then we present a modification of the encoder to account for such a difference.

A. Wireless transmission method

There exists many well-known transmission techniques that can be used to transmit the information symbols $\delta_k \in \{-1, 0, 1\}$ at the rate $1/T_s$, see for example [13] for a complete description. One possibility is to use a ternary RZ line code associated to an amplitude shift keying modulation (ASK). Let us describe the corresponding transmission and the reception stages.

1) *Transmission:* The transmission stage can be divided into two different steps: the line coding and the modulation.

a) *Line coding:* We have first of all to build a baseband, low frequency signal, also called line code. It writes:

$$s_{LF}(t) = \sum_k \delta_k h_e(t - kT_s) \quad (10)$$

with h_e an emission filter whose aim is to precise to time-frequency localization of the transmitted signal. For example, $h_e(t)$ can be a simple rectangular window with duration T_s . In this case, we get the so called NRZ (Non Return to Zero) line code. The rectangular window can also have a duration $\tau < T_s$, which corresponds to RZ (Return to Zero) line code¹. This is the choice adopted in this work. This signal, $s_{LF}(t)$, could be transmitted through a wired line, but not through a transmission channel. Indeed, wireless transmission is possible only for high frequency signal (HF), whereas $s_{LF}(t)$ is low frequency by construction. It is worthwhile mentioning that the spectral properties of this signal are fixed by $h_e(t)$. In practice, the spectrum is never perfectly bounded, but we can nevertheless make the approximation that there exists an upper maximal frequency $f_{\max} : |H_e(f)| \ll 1$ for $|f| > f_{\max}$.

b) *Modulation:* The modulation step consists in transposing the low frequency signal $s_{LF}(t)$ around a carrier frequency f_0 in order to transmit it through radio waves. We have chosen to use an amplitude modulation for its ease of implementation and also in order to take advantage of the fact that the signal will be equal to zero when $\delta_k = 0$, reducing the energy used. Then, the corresponding high frequency modulated signal simply writes :

$$s_{HF}(t) = s_{LF}(t) \cos(2\pi f_0 t)$$

The modulated signal spectrum is then concentrated around $-f_0$ and f_0 , precisely in the intervals $[-f_{\max} - f_0; +f_{\max} - f_0]$

¹Classically, RZ and NRZ line codes rather use binary symbols, but they of course can be extended to the case of ternary symbols.

and $[-f_{\max} + f_0; +f_{\max} + f_0]$. Thus, we must have $f_0 > f_{\max}$, otherwise these two intervals overlap. In practice, $f_0 \gg f_{\max}$.

2) *Reception*: We will suppose that the received signal is exactly equal to the transmitted signal $s_{\text{HF}}(t)$, which is obviously not the case in reality. We will so neglect synchronization issues, noise and any distortions due to the transmission channel. The reception stage is dual to the transmission stage. Thus, it comprises a demodulation step followed by a decoding and a detection step.

a) *Demodulation*: We first have to demodulate the received high frequency signal $s_{\text{HF}}(t)$, so that we will recover $s_{\text{LF}}(t)$. The optimal solution consists in multiplying $s_{\text{HF}}(t)$ by the same carrier as the one used at the modulation step, and then low-pass filtering the obtained signal. Indeed, we have

$$s_{\text{HF}}(t) \cos(2\pi f_0 t) = \frac{1}{2} \underbrace{s_{\text{LF}}(t)}_{|f| < f_{\max}} + \underbrace{s_{\text{LF}}(t) \cos(2\pi 2f_0 t)}_{|f-2f_0| < f_{\max} \text{ or } |f+2f_0| < f_{\max}}$$

and we thus deduce that $s_{\text{LF}}(t)$ can be recovered by low pass filtering of $s_{\text{HF}}(t) \cos(2\pi f_0 t)$, providing that this filter $H_{\text{LF}}(\nu)$, is such that :

- $H_{\text{LF}}(\nu) = 2$ pour $|\nu| < f_{\max}$;
- $H_{\text{LF}}(\nu) = 0$ pour $|\nu| > -f_{\max} + 2f_0$.

That is why $2f_0$ has to be greater enough than $f_{\max}$, otherwise the low-pass filter will not be practically feasible.

b) *Decoding*: Let us denote $\hat{s}_{\text{LF}}(t)$ the signal obtained after low pass filtering. If the low pass filtering is perfect and if $s_{\text{LF}}(t)$ is completely spectrally concentrated in $[-f_{\max}; f_{\max}]$, then $\hat{s}_{\text{LF}}(t) = s_{\text{LF}}(t)$. Otherwise, in a real case, a slight error $\varepsilon(t)$ occurs : $\hat{s}_{\text{LF}}(t) = s_{\text{LF}}(t) + \varepsilon(t)$. This slight error is not problematic in fact. Indeed, what is important is not to reconstruct exactly $s_{\text{LF}}(t)$, but only to recover the δ_k symbols. It can be shown that the optimal receiver consists in filtering by $h_e(-t)$ and then sampling at the instant kT_s :

$$\bar{\delta}_k = \int_{-\infty}^{\infty} \hat{s}_{\text{LF}}(t) h_e(t - nT_s) dt = [\hat{s}_{\text{LF}}(t) * h_e(-t)]_{t=kT_s} \quad (11)$$

The problem is that $h_e(-t)$ is not a causal filter. That is why, even for a perfectly reliable channel without noise, a delay need to be introduced. Let us denote τ the duration of $h_e(t)$. Then, $h_e(\tau - t)$ is causal and

$$\bar{\delta}_k = [\hat{s}_{\text{LF}}(t) * h_e(\tau - t)]_{t=kT_s + \tau}$$

Thus, the symbol δ_k transmitted at the instant kT_s can only be received at the instant $kT_s + \tau$, that is to say with a delay τ . Moreover, this delay is equal to the duration of $h_e(t)$. Thus, the lower is the duration of $h_e(t)$, the lower will be this delay. But a lower duration also implies a greater spectral occupancy. It is worthwhile noting that if there were no noise, this filtering would not be necessary, and therefore there no demodulation delay.

c) *Detection*: At least, after the computation of an estimation $\bar{\delta}_k$ of the symbol transmitted at the instant kT_s , we can take advantage of the fact that $\delta_k \in \{-1, 0, 1\}$ in using a threshold non linearity with the following detection rule:

$$\hat{\delta}_k \triangleq \begin{cases} -1 & \text{if } \bar{\delta}_k < -0.5 \\ 0 & \text{if } |\bar{\delta}_k| < 0.5 \\ 1 & \text{if } \bar{\delta}_k > 0.5 \end{cases}$$

to get the final estimated $\hat{\delta}_k$.

In conclusion, it is worthwhile emphasizing that :

- we must have $H_{\text{LF}}(\nu) = 2$ for $|\nu| < f_{\max}$ and $H_{\text{LF}}(\nu) = 0$ for $|\nu| > -f_{\max} + 2f_0$, hence $f_0 \gg f_{\max}$;
- if δ_k is transmitted at the instant kT_s , it is received at the instant $kT_s + \tau$. Therefore:

$$\hat{\delta}(kT_s) = \delta(kT_s - \tau), \quad \forall k \in \mathbb{Z}^+, \tau > 0 \quad (12)$$

B. Proposed modification for latency compensation

We have just seen that the transmission process introduces a delay τ . Therefore, we can not assume anymore that $\hat{\delta}_k = \delta_k$ as in section IV. Nevertheless, we will now see that it is possible to build a predictor with modified time-horizon at the decoder side, so that the stability conditions remain unchanged.

Without loss of generality, let us assume that² $\tau = T_s/2$. This implies that the information needs to be processed with higher rate, and hence the digital version of the decoder needs to compile with this new sampling rate. To this aim, let us introduce the new delay operator z^{-1} in this new time-base, as: $v(t)z^{-1} = v(t - T_s/2)$. We have then the following relation $z^{-2} = q^{-1}$, and according to (12) we also have:

$$\hat{\delta}_k = z^{-1} \delta_k$$

Thus, the transmission latency can be assimilated to a pure delay equal to z^{-1} . The idea of the modification proposed can be explained by first rewriting the encoder equation in this new time basis (although there is no need to make any change of equation nor to modify the time basis of the clock at the encoder side). This gives,

$$\hat{x}_k = W(z^{-2}) \frac{\Delta}{1 - z^{-2}} \delta_k$$

and noticing that if the previous decoder structure is modified as:

$$\begin{aligned} \hat{x}_k &= W(z^{-2}) \frac{\Delta z^{-1}}{1 - z^{-2}} \hat{\delta}_k = W(z^{-2}) \frac{\Delta}{1 - z^{-2}} \delta_k \\ &= W(q^{-1}) \frac{\Delta}{1 - q^{-1}} \delta_k \end{aligned}$$

where the last expression is obtained by using the fact that $\hat{\delta}_k = z^{-1} \delta_k$. Then, if the encoder and the decoder are initialized equally, the solutions and the associated error equation are similar to the ones presented in section IV. As a consequence, the stability properties follow the results in Theorem 1.

²the principle of our proof is valid for any $\tau < T_s$

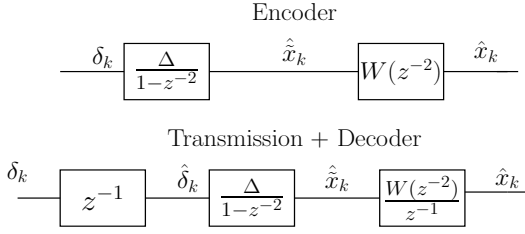


Fig. 3. Decoder and encoder equations.

VI. SIMULATION RESULTS

We consider the following simple system

$$\frac{B(q^{-1})}{A(q^{-1})} = \frac{bq^{-1}}{1 - aq^{-1}} \quad (13)$$

The controller is: $u_k = k\hat{x}_k + \gamma r_k$ obtained from the closed-loop specification given by $A_{cl} = (1 - a_{cl}q^{-1})$, $k = -\frac{a - a_{cl}}{b}$ and $\gamma = 1 - a_c$. Parameter used in simulations are: $a = 1.1$, $b = 1$, $a_c = 0.5$, $T_s = 0.1$ (sec), $\Delta = 0.02$, $x_0 = 0$ and $\hat{x}_0 = -0.01$ so $\tilde{x}_0 = 0.01$.

With $\rho_1 = \frac{1 - \frac{a - a_c}{2}}{1 - a_c} \Delta = 0.028$, we have $\tilde{x}_0 < \rho_1$ and $a < 1 + a_c = 1.5$ so theorem 1 applies.

The upper figure 4, shows that x_k can track the reference r_k with a certain error function of the quantification Δ . Figure 4 (bottom) displays the distribution of δ_k : we can see that when the reference is constant (as the output) the encoder sends a substantially large number of $\delta_k = 0$. As the the encoder has no energy consumption during the $\delta_k = 0$ phases, an important saving in energy is obtained at the sensor side. This saving is much more important than the ones that can be obtained by using the standard two-state Δ -modulation ($\delta_k = \pm 1$) strategy.

VII. CONCLUSIONS

In this paper we have investigated the possibility to use 3-level coding alphabet in the context of networked controlled systems. The main motivation has been to explore the benefits in terms of energy savings (energy-aware) in the context of low-energy sensors. We have proposed to use a coding strategy with the ability to quantify and to differentiate stand-still signal events, from changes in the source (level crossing detector). The stand-still signal event results then in a modulation strategy with a low energy carrier (sleep mode), whereas the changes of levels are modulated with enough energy (wake-up mode).

We have studied the impact of the wireless RZ amplitude modulation strategy in connection with the stability of the system. It has been shown that this modulation introduced an arbitrarily small latency, but that it can be compensated at the decoder side, by accommodating the predictor horizons to this delay which is known and fixed by the user. In that way, the same stability condition as in the case of ideal transmission are preserved.

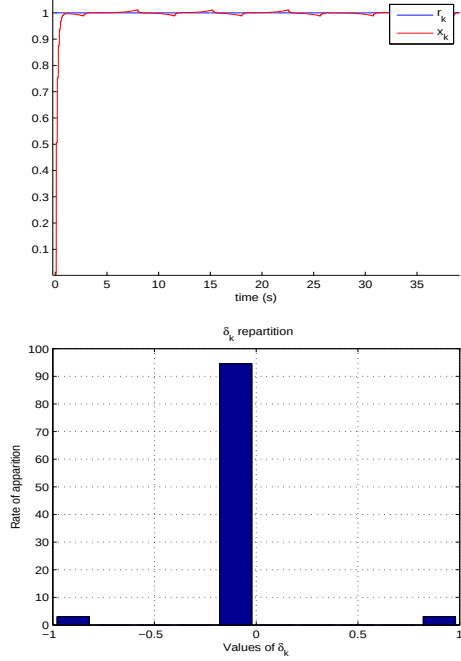


Fig. 4. Time evolution of x_k and r_k (upper). Event distribution of δ (bottom).

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