Spin-Squeezing and Light Entanglement in Coherent Population Trapping

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We show that high squeezing and entanglement can be generated at the output of a cavity containing atoms interacting with two fields in a Coherent Population Trapping situation, on account of a non-linear Faraday effect experienced by the fields close to a dark-state resonance in a cavity. Moreover, the cavity provides a feedback mechanism allowing to reduce the quantum fluctuations of the ground state spin, resulting in strong steady state spin-squeezing.

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Coherence-mediated effects in atomic media have received considerable attention in connection with magnetometry [1], coherent population trapping [2], electromagnetically induced transparency [3], slow-light [4], non-linear optics [5] or atomic spin-squeezing [6]. When two fields of about the same strength interact resonantly with three-level Λ-type atoms - situation commonly referred to as Coherent Population Trapping (CPT) - the atoms are pumped into a superposition of the ground state levels, which is a state with maximum coherence. Close to the CPT (or dark-) resonance the fields experience a strong dispersive effect, but little absorption [3]. Consequently, several schemes have been studied which take advantage of this strong non-linearity to generate quantum correlations and squeezing in the fields under EIT or CPT conditions [7, 8, 9].

Following the recent squeezed and entangled light states generation with cold atoms [10, 11], we study in this Letter the interaction of atoms inside an optical cavity with two field modes close to a dark-resonance. We first show that a multistable behavior for the intracavity light may occur close to the CPT resonance on account of a non-linear polarization self-rotation effect experienced by the light [12, 13]. Such polarization instabilities have been observed with cold atoms [10] and thermal vapor cells [13]. We then calculate the field noise spectra and predict that strong correlations exist between the fields exiting the cavity and that squeezing and entanglement can be generated close to the switching threshold and for a wide range of parameters. In contrast with the experiments of Refs. [10, 11, 14] the squeezing and the entanglement are not deteriorated by excess atomic noise due to optical pumping processes and could be efficiently generated either with cold atoms or thermal vapor cells. Last, we show that under appropriate conditions on the cavity induced feedback the ground state atomic spin fluctuations may also be strongly squeezed, and subsequently read out using techniques developed within the context of quantum memory [15].

We consider N Λ-like atoms with ground-states 1 and 2, interacting with two modes of the field, A_1 and A_2. To simplify the discussion we turn to the symmetrical case of incident fields with equal power and will choose the parameters so that the Rabi frequencies of both transitions are equal: $\Omega_i = \Omega$ $(i = 1, 2)$ and close to one-photon resonance. This situation corresponds to Coherent Population Trapping, since the atoms are pumped into a superposition of levels 1 and 2 - the so-called *dark-state* - which is decoupled from the fields [2]. For $\Omega_1 = \Omega_2$, this dark-state corresponds to a state with maximum ground state coherence: $|D\rangle = (|1\rangle - |2\rangle)/\sqrt{2}$. Although this is not essential, let us note that, if levels 1 and 2 are Zeeman sublevels, the incident field can then be considered as being linearly polarized. As we will see later this facilitates the physical discussion and provides a simple picture in terms of the Stokes polarization vector and the collective spin formalism.

If the fields are symmetrically detuned with respect to the atomic resonance: $\Delta_2 = -\Delta_1 = \delta$, and for opposite cavity detunings $\Delta_{c1} = -\Delta_{c2} = \kappa\varphi$, the intracavity intensities are indeed symmetrical with respect to each mode and satisfy

$$I^{in} = I\left[(1+A)^2 + (\varphi - \phi_{nl})^2\right] \quad (1)$$

$$A = C\frac{\bar{\delta}^2}{I^2 + \bar{\delta}^2 + I\bar{\delta}^2 + \bar{\delta}^4} \quad (2)$$

$$\phi_{nl} = C\frac{\bar{\delta}(I - \bar{\delta}^2)}{I^2 + \bar{\delta}^2 + I\bar{\delta}^2 + \bar{\delta}^4} \quad (3)$$

with $I = \Omega^2/\gamma^2$, $\bar{\delta} = \delta/\gamma$, γ the optical dipole decay rate, κ the cavity bandwidth and $C = g^2N/T\gamma$ the usual cooperativity parameter, g being the atom-field coupling constant and T the intensity transmission of the coupling mirror. In the vicinity of the CPT resonance ($\delta = 0$), levels 1 and 2 are equally populated, and the ground state coherence is real and maximal: $\langle J_x\rangle \simeq -N/2$. As illustrated in Fig. 1, the absorption of the fields is drastically reduced within a narrow transparency window, while there is a strong change in the dispersion.

Fully on resonance the medium is rendered transparent in steady state for both fields, so that, if the input fields are in a coherent state, the output fields will also be uncorrelated. However, in the vicinity of such a dark resonance, Eqs. (2-3) show that the non-linearity and absorption A of the medium are given by

$$A \sim \frac{C\delta^2\gamma^2}{\Omega^4}, \qquad \phi_{nl} \sim \frac{C\delta\gamma}{\Omega^2} \quad (4)$$

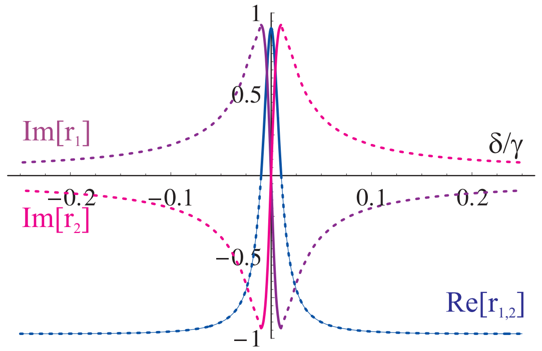

FIG. 1: Real and imaginary parts of the reflectivity coefficients of the cavity when the two-photon detuning δ is varied. Parameters: $C = 100$, $I = 1$, $\varphi = 0$. The dashed parts corresponds to unstable solutions for the linearly-polarized light.

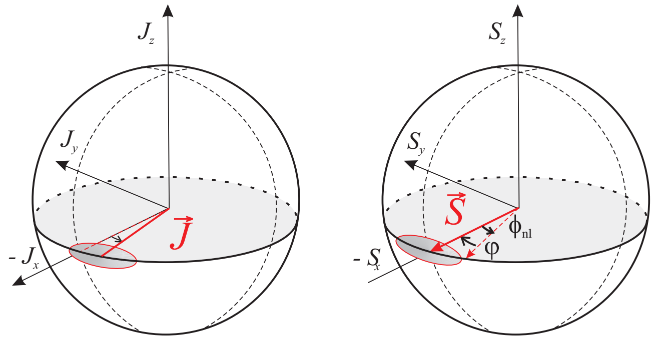

FIG. 2: Self-rotation induced on the field close to the CPT resonance: a non-zero detuning δ causes the spin to rotate in the equatorial plane of the Bloch sphere by an angle proportional to δ. Because of the Faraday effect, the Stokes vector also tends to rotate by an angle ϕ_{nl} proportional to δ in the equatorial plane of the Poincaré sphere, but the detuned cavity brings it back along x. The ellipsoids represent the quantum fluctuations of the Bloch and Stokes vectors.

when $\Omega \gg \gamma, \delta$. These quantities are to be compared with the absorption and Kerr non-linearity in a two-level system [10, 16]

$$A \sim \frac{C\gamma^2}{2\Delta^2}, \qquad \phi_{nl} \sim \frac{C\gamma\Omega^2}{\Delta^3} \tag{5}$$

($\Delta \gg \gamma, \Omega$). For squeezing or entanglement generation the figure of merit of both systems can be evaluated qualitatively by comparing the non-linearity to absorption ratio ϕ_{nl}/A. In both cases these ratios take on a similar form: $(\phi_{nl}/A)_{CPT} = \Omega^2/\delta\gamma$ and $(\phi_{nl}/A)_{Kerr} = 2\Omega^2/\Delta\gamma$, but, the sharpness of the dark resonance allows to reach a strong non-linearity as well as little absorption with much lower intensities [5].

A first consequence of this non-linearity is the appearance of a multistable behavior for the intracavity light [12]. As shown in Fig. 1 the symmetrical solution - always stable for $\delta = 0$ - is unstable above a certain threshold when the two-photon resonance condition is no longer ensured. From Eq. (1) the stability condition corresponds to

$$\delta \leq \delta_s = \sqrt{1+\varphi^2}\frac{\Omega^2}{\gamma C} \qquad (C \gg 1, I \gg \bar{\delta}). \tag{6}$$

If one considers the A_1 and A_2 modes to be circularly-polarized, this threshold can be traced to a coherence-induced non-linear *self-rotation* [12, 13]. For the chosen Rabi frequencies the incident field is linearly-polarized along the y-axis, which means that the Stokes vector is aligned along $-Ox$ in the Poincaré sphere (Fig. 2). On resonance ($\delta = 0$) the ground state spin in the Bloch sphere is parallel to the Stokes vector : $\langle J_x\rangle = -N/2$, $\langle J_y\rangle = \langle J_z\rangle = 0$. If one lifts the ground state sublevel degeneracy (with a longitudinal magnetic field for instance), the intracavity fields experience opposite non-linear phase-shifts as can be seen from Fig. 1. The Stokes vector would thus tend to rotate in the equatorial plane of the Poincaré sphere under the influence of this non-linear Faraday effect (Fig. 2), but, if the phase-shift stays smaller than the cavity and atomic losses then the Stokes vector stays along the x-axis. Because of the Faraday effect the ground state spin rotates in the equatorial plane by a small angle proportional to δ at first order:

$$\langle J_x\rangle \simeq -\frac{N}{2}, \quad \langle J_y\rangle \simeq \frac{N}{2}\frac{\delta\gamma}{\Omega^2}. \tag{7}$$

We now turn to the modification of the outgoing field noise spectra in the vicinity of the CPT resonance. We calculate these spectra in a standard fashion by linearizing the equations around the semi-classical state corresponding to a working point in the stable range defined previously. The noise spectrum $S_{X_\theta}(\omega)$ of any quadrature $X_\theta = Ae^{-i\theta} + A^\dagger e^{i\theta}$ can be obtained from the atom-field covariance matrix [17]. To examine the occurrence of squeezing generated by the system we have represented in Fig. 3 the minimal noise spectra $S^*(\omega) = \min_\theta S_{X_\theta}(\omega)$ of different modes versus the analysis frequency. For a good choice of the interaction parameters, substantial squeezing can be observed in the $A_{1,2}$ modes, as well as in the "dark" and "bright" linearly-polarized modes

$$A_x = (A_2 - A_1)/\sqrt{2}, \quad A_y = -i(A_1 + A_2)/\sqrt{2}. \tag{8}$$

Since $\langle A_x\rangle = 0$, the "x"-polarized component of the field exiting the cavity is in a squeezed vacuum state, or, equivalently, in a *polarization-squeezed* state [18], as the quantum noise of an orthogonal component of the Stokes vector is reduced (Fig. 2).

The fact that two modes with orthogonal polarization are squeezed at the output of the cavity is a signature of quantum correlations existing between orthogonal modes, and therefore of entanglement. A general method to find out the modes possessing the highest amount of EPR-type correlations has been outlined in Ref. [19], and

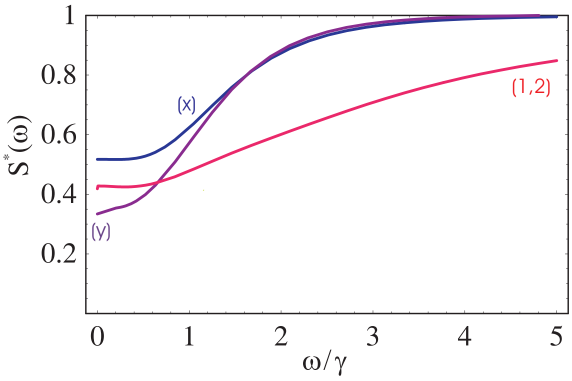

FIG. 3: Squeezing spectra of the circularly- and linearly-polarized modes, $A_{1,2}$ and $A_{x,y}$ [as defined by Eq. (8)]. Parameters: $C = 100$, $\kappa = 2\gamma$ $\bar{\delta} = 1$, $\varphi = 1$, $I = 144$.

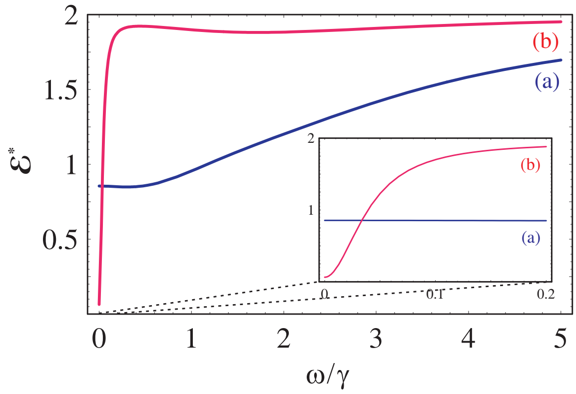

FIG. 4: Maximal entanglement $\mathcal{E}^*$ that can be obtained by adequately mixing the outgoing fields versus sideband frequency ω/γ. (a) Parameters as in Fig. 3 (b) $C = 1000$, $\kappa = 2\gamma$, $\bar{\delta} = 0.1$, $\varphi = 2$, $I = 49$.

experimentally tested in Ref. [11]. This method is based on minimizing the quantity

$$\mathcal{E}_{a,b} = \left[\Delta(X_a - X_b)^2 + \Delta(Y_a + Y_b)^2\right]/2 \qquad (9)$$

under unitary transforms on modes a, b (rotations of the polarization basis), where $X = X_{\theta=0}$ and $Y = X_{\theta=\pi/2}$ are the standard amplitude and phase quadrature operators. The orthogonal odes a and b are in a non-separable state if $\mathcal{E}_{a,b} < 2$ [20]. The highest amount of quantum correlations, $\mathcal{E}^* = \min_{a,b}\mathcal{E}_{a,b}$, that can be produced by the system is plotted versus frequency in Fig. 4 and is of the order of 3 dB for typical experimental parameters. Numerical simulations show that the bandwidth over which the squeezing or the entanglement are observed depends both on the cavity bandwidth κ and the atomic detuning δ. For values of the detuning much smaller than κ and close to the threshold value, the entanglement bandwidth is given by the CPT width δ_s. Entanglement is thus larger at zero-frequency and substantially increases with the cooperativity C. Noise reductions of 15 dB can be reached for $C \sim 1000$ and $\delta \ll \gamma$ (inset in Fig. 3). However, squeezing - or entanglement - is produced in a much smaller bandwidth. Note also that the non-linear Faraday rotation considered here is very different from the linear self-rotation effect of Refs. [10, 14] and does not originate from optical pumping processes. The squeezing is therefore not deteriorated by excess atomic noise, and much stronger squeezing can thus be obtained.

Let us now focus on the atomic fluctuations. For a two-level atom ensemble polarized along the x-axis *spin-squeezing* is associated to the noise reduction in a spin component in the Oyz-plane below the standard quantum limit given by $\Delta J_y\, \Delta J_z \geq |\langle J_x\rangle|/2$, and corresponds to the establishment of quantum correlations between individual spins. The occurrence of spin-squeezing can be examined by numerically calculating the minimal variance of spin components in the plane orthogonal to the mean spin direction. We observe for some values of the cavity detuning and close to the threshold strong spin-squeezing in the population difference J_z. In order to gain a physical insight into the process responsible for spin-squeezing we assume a small detuning so that the spin is mostly polarized along x: $\langle J_x\rangle \simeq -N/2$, and choose the working point close to the threshold: $\alpha = \delta_s/\delta \gtrsim 1$ in order to calculate the fluctuations of J_z. Close to the CPT resonance there are two dominant contributions to the noise of the population difference J_z: first, fluctuations of the optical dipole P_x induced by the *dark* mode A_x, secondly, the projection of the *bright* mode A_y fluctuations due to the Faraday rotation

$$\delta J_z \propto i\Omega\, \delta P_x - a\, \delta A_y + h.c. + noise, \qquad (10)$$

with $a = gN\delta/2\sqrt{2}\Omega$. For small detunings, the population difference response time - given by (12) - is much smaller than the optical dipole or the intracavity field evolution times. Moreover, for a strongly detuned cavity ($\varphi \gg 1$), the *bright* mode fluctuations δA_y will be the sum of fluctuations due to the *dark* dipole P_x and to the incident *dark* mode A_x^{in} (damped by a factor φ)

$$\delta A_y \propto ib\, \delta P_x + c\, \delta A_x^{in}, \qquad (11)$$

with $b = 2\sqrt{2}g/T\varphi$ and $c = 2/\varphi\sqrt{T}$. It is therefore possible for the fluctuations of the dipole induced by the cavity feedback to compensate the "natural" fluctuations in (10) when $\Omega \simeq ab$. This condition exactly corresponds to the threshold condition (6) when $\varphi \gg 1$. After adiabatically eliminating the optical dipole and the field, it can be shown that the fluctuations of J_z are proportional to the fluctuations of the incident *dark* mode amplitude quadrature X_x^{in}, damped by the cavity detuning. The atomic noise spectrum is Lorentzian-shaped with a width γ_z, given by

$$\gamma_z = \delta\sqrt{1+\varphi^2}\left(\alpha - \alpha^{-1}\right). \qquad (12)$$

γ_z clearly vanishes at the threshold ($\alpha = 1$) as the system becomes unstable at the bifurcation point. The normal-

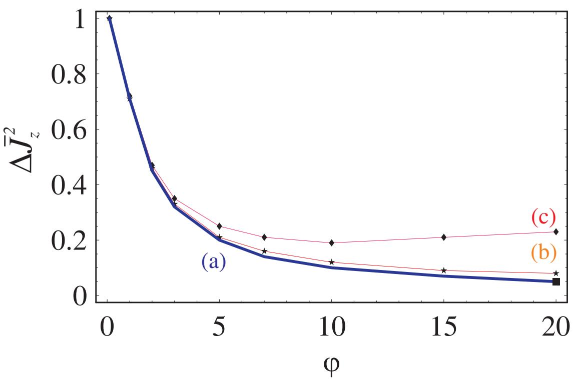

FIG. 5: $\Delta\bar{J}_z^2$ versus the cavity detuning φ for $2\delta = 0.01\gamma$: (a) Analytical (14), (b) $C = 1000$, (c) $C = 100$. α is chosen to optimize the squeezing for each value of the cavity detuning.

ized variance in the population difference is then

$$\Delta\bar{J}_z^2 = \frac{\Delta J_z^2}{N/4} \simeq \frac{(\alpha\sqrt{1+\varphi^2}-\varphi)^2+1}{(1+\varphi^2)(\alpha^2-1)}. \qquad (13)$$

Minimizing this quantity with respect to α yields the optimal spin-squeezing

$$\Delta\bar{J}_z^* \simeq \frac{1}{\sqrt{1+\varphi^2}} \qquad (C \gg 1,\ \delta \ll \gamma). \qquad (14)$$

This simple result expresses that the fluctuations of the ground state population difference are indeed damped by the cavity feedback. Because of the Faraday rotation fluctuations of the *bright* mode A_y are projected onto the transverse spin components J_y and J_z, the fluctuations of which are either amplified or reduced depending on the cavity detuning (Fig. 2). A large spin-squeezing can thus be obtained close to the CPT resonance. The simple expression of Eq. (13) and the numerical simulation results are shown in Fig. 5; an excellent agreement is found as long as $C\bar{\delta} \gg 1$. When this condition is not satisfied spontaneous emission noise can no longer be neglected. We have nevertheless observed that smaller, but still significant, spin-squeezing can be obtained for larger values of $\bar{\delta}$.

In conclusion, we have presented a cavity scheme based on an atomic coherence induced non-linear Faraday effect to generate strong squeezing and entanglement of light fields. Moreover, this scheme also predicts strong quantum correlations between the light and atomic variables, resulting in spin-squeezing. Note that the spin-squeezing mechanism presented here is quite different from other schemes based upon Faraday rotation [21, 22], since it relies on both a strong non-linear interaction as well as a constructive cavity feedback. We can draw a parallel with the experiments of Ref. [22] in which the Faraday effect-induced fluctuations of a field that has propagated through a cold atom cloud is fed back to the atoms in order to actively control the atomic fluctuations. A major difference is that the feedback is automatically provided by the cavity. Last, the spin-squeezed state generated could be probed at a later time by switching back on the fields and performing an adequate measurement of the fluctuations of the Stokes parameter S_z [15].

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