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Crumpled glass phase of randomly polymerized membranes in the large d limit

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Abstract. — Tethered phantom membranes with quenched disorder in the internal preferred metric are studied in the limit of large embedding space dimension $d \to \infty$. We find that the instability of the flat phase previously demonstrated via ϵ-expansion is towards a spin-glass-like phase which we call the crumpled glass phase. We propose a spin-glass order parameter that characterizes this phase and derive the free energy which describes the crumpled, flat and crumpled glass phases of the disordered membrane. The crumpled glass phase is described by local tangents which vanish on average, but display a nonzero Edwards-Anderson spin-glass order parameter. From the saddle point equations at large d we obtain the equation of state, phase diagram and the exponents characterizing these phases. We estimate the effects of the higher order corrections in the $1/d$ expansion by utilizing previous results for pure membranes. We use Flory arguments to calculate the wandering exponents and discuss the relevance of self-avoidance in the crumpled glass phase.

1. Introduction.

Recently there has been considerable interest in polymerized surfaces which are two-dimensional generalizations of linear polymers [2, 3]. Unlike polymers these higher dimensional generalizations are expected to exhibit both the crumpled high temperature phase and a flat low temperature phase with a second order phase transition between the symmetric and broken phases. The flat phase is described by singular elastic moduli renormalized by the nonlinear interactions between in-plane and out-of-plane fluctuations of the random surface [4-6]. The existence of the crumpled and flat phases has been verified in computer simulations of a non-self-avoiding "phantom" membrane [7-9]. The crumpled phase has also been recently seen in Monte Carlo simulations of self-avoiding tethered surfaces modelled by impenetrable flexible

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plaquetes [10]. A red blood cell with spectrin attached to the lipid cell wall is an example of a biological tethered surface and can be described by these theories [11]. Inorganic examples of tethered surfaces include graphite oxide sheets in an appropriate solvent [12] and the "rag" sheetlike structures found in MoS_2 [13]. For other experimental realizations, see reference [3].

The first study that investigated the effects of internal disorder on these random surfaces was done by Nelson and Radzihovsky[1] . They considered the effects of quenched impurities, dislocations and disclinations on these structures and modelled the disorder by random local fluctuations in the ground state internal metric. Within the $\epsilon = 4 - D$ expansion it was found that the impurities and dislocation (uncorrelated) disorder leads to a $T \to 0$ instability of the flat phase. For $D \leq 4$ this instability results from the addition of any amount of disorder, while for $D > 4$ there is a finite threshold at low temperatures. For *finite* temperatures the flat phase was found to be stable to the addition of weak impurity disorder. The disclination (long-range correlated) disorder, however, was found to lead to an instability toward strong disorder region at *all* temperatures.

Although the instability was to a strong disorder region that lies outside the range of validity of the ϵ expansion there were some indications that it leads to a "spin-glass" phase [14]. One such indication was the softening of the renormalized bending rigidity κ_R as $T \to 0$ leading to many different nearly degenerate configurations characteristic of a glassy phase. Because of the frustration induced by the random preferred metric, the bending energy cannot be minimized simultaneously with the elastic energy resulting in a random buckling of the membrane out of the plane. These properties were argued to lead to Edwards-Anderson Ising-like spin-glass [15-19] with up and down puckers of the membrane playing the role of spins with random interactions. This type of spin-glass phase (if it exists) would probably appear at intermediate strength of disorder, and would describe a *roughened* membrane with a nonvanishing average tangent field. It is thus appropriate to think of this phase as a *flat* spin glass. Work is currently in progress to investigate its existence and properties.

The fact that quenched internal disorder can drastically modify the thermodynamics of polymerized membranes was recently demonstrated experimentally by Mutz, Bensimon and Brienne [20]. They observed that partially (heterogeneous) polymerized vesicles undergo upon cooling a transition to a folded rigid stucture. They interpreted their experiment as an evidence for a transition towards a crumpled spin-glass-like state. Although it is not yet clear which model is appropriate to describe their experiment, it provides a strong motivation to look for possible models of crumpled glass phases.

A low temperature instability toward strong disorder was also found in the random-axis model [21] which is somewhat analogous to the present problem with the unit normal to the surface playing the role of the spin. For these systems an ϵ expansion led to flow diagrams very similar to the ones found for disordered membranes in reference [1] with a low temperature instability toward strong disorder region. Further studies of this problem have led to the identification of the new phase of a random-axis model with an isotropic spin-glass phase [22-27]. These results on spin systems were in part the original motivation for this work and suggested to us that the flat phase of the membrane is unstable to a crumpled spin-glass phase [28].

Here we propose that the model of reference [1] exhibits a *crumpled* spin-glass phase characterized by a vanishing average tangent field $\overline{\langle \partial_\alpha r_i \rangle}$ but with a nonzero crumpled spin-glass order parameter $\overline{\langle \partial_\alpha r_i \rangle \langle \partial_\beta r_j \rangle}$. We investigate this conjecture by utilizing a $1/d$-expansion [29,30].

In the limit of large embedding dimension, $d \to \infty$, the *pure* model can be solved exactly. [6] However, for a *disordered* membrane even in the $d \to \infty$ limit the exact solution of the spin-glass phase seems intractable. The difficulty arises from the tensor structure of the spin-glass order parameter which leads to a problem of matrix field theory, a notoriously difficult problem. To make progress we make an approximation in the spirit of mean-field theory in which we ignore the fluctuations in the tensor spin-glass order parameter. We note however that since we consider large d we can still integrate out exactly the fluctuations of the tangent vectors, thus obtaining the tree level effective free energy for the spin-glass order parameter. Within this theory we find an Edwards-Anderson-like crumpled spin-glass phase.

This paper is organized as follows. In section 2 we introduce the model of reference [1] for the disordered tethered phantom membrane. Using replica "trick" we derive the effective free energy for this system to the leading order in $1/d$ with the approximation alluded to in the previous paragraph. The saddle point equations, within the replica symmetric ansatz, are obtained in section 3 and are analyzed in section 4 for the pure and the disordered membrane. We determine the phase diagrams and calculate the critical exponents describing these phases and the transitions. In section 5 the results of the calculations are discussed. We consider the effects of the higher order $1/d$ corrections to our results. Using the Flory type of arguments we estimate the radius of gyration exponent in the crumpled glass phase. It determines the scaling of the membrane's size with the internal dimension, inside the embedding space. The relevance of the self-avoiding interaction is also considered. In section 6 we summarize our results and pose many interesting unaswered questions as the subjects for future investigations.

2. Effective free energy in the large d limit.

We use the disordered free energy with the disorder introduced through the local deviations $\delta c(\mathbf{x})$ in the ground-state metric, as was first proposed in reference [1]. The probability of a particular configuration for fixed disorder configuration is proportional to $\exp(-F_c[\vec{r}]/T)$, where

$$F_c[\vec{r}] = \int \mathrm{d}^D x \left[\frac{1}{2}\,\kappa d|\nabla^2\vec{r}|^2 + \frac{1}{4}\,\mu d\,\{\partial_\alpha\vec{r}\cdot\partial_\beta\vec{r} - \delta_{\alpha\beta}[1+2\delta c(\mathbf{x})]\}^2 + \frac{1}{8}\,\lambda d\,\{\partial_\gamma\vec{r}\cdot\partial_\gamma\vec{r} - D[1+2\delta c(\mathbf{x})]\}^2\right] \tag{2.1}$$

Above, κ is the bending rigidity, and μ and λ are the elastic Lamé coefficients of the membrane. We will restrict our considerations to impurity (uncorrelated) disorder and take $\delta c(\mathbf{x})$ to be a zero mean Gaussian quenched random field, with probability distribution $P[\delta c(\mathbf{x})] \propto \exp\left[-\frac{1}{2\sigma}\int \mathrm{d}^D x\,\delta c^2(\mathbf{x})\right]$. It describes random dilations and compressions in the locally preferred metric, $\partial_\alpha\vec{r}\cdot\partial_\beta\vec{r} = \delta_{\alpha\beta}[1+2\delta c(\mathbf{x})]$, due to disorder. In the above we have rescaled the elastic moduli by letting μ, $\lambda \to \mu d$, λd, to obtain sensible and nontrivial results in the limit $d \to \infty$.

To simplify the calculation we introduce an auxilary field $\chi_{\alpha\beta}$ and perform a Hubbard-Stratanovich transformation on the quartic part of the free energy [31-30].

$$F_c[\vec{r},\chi_{\alpha\beta}] = \int \mathrm{d}^D x \left[\frac{d}{2}\,\kappa|\nabla^2\vec{r}|^2 + \frac{d}{2}\,\chi_{\alpha\beta}\,(\partial_\alpha\vec{r}\cdot\partial_\beta\vec{r} - \delta_{\alpha\beta}) - \alpha\frac{d}{2}(\chi_{\alpha\beta})^2 - \beta\frac{d}{2}(\chi_{\alpha\alpha})^2 - d\frac{(\lambda D + 2\mu)}{2}\delta c\,\partial_\alpha\vec{r}\cdot\partial_\alpha\vec{r}\right] \tag{2.2}$$

In the above $\alpha = 1/(2\mu)$ and $\beta = -\lambda/(2\mu(2\mu + D\lambda))$.

The disorder-averaged effective free energy F_{eff} is given by the average of the logarithm of the partition function for each configuration of disorder, Z_c, with the weight $P[\delta c]$

$$-\frac{1}{T}F_{\text{eff}} = \int \mathcal{D}\delta c P[\delta c] \ln Z_c \ , \tag{2.3a}$$

$$Z_c = \int \mathcal{D}\vec{r} \int \mathcal{D}\chi_{\alpha\beta} \exp\{-\frac{1}{T}F_c[\vec{r}, \chi_{\alpha\beta}]\} \tag{2.3b}$$

We use the replica formalism [15] to treat the disorder. As usual we introduce n copies of fields $\vec{r}$ and $\chi_{\alpha\beta}$ labeled by the replica index a with the total free energy given by the replicated version of the free energy equation (2.2). The relation to the original nonreplicated free energy is established through the identity

$$\ln Z_c = \lim_{n \to 0} \frac{Z_c^n - 1}{n} \tag{2.4}$$

Assuming that we can interchange the thermodynamic limit and the limit $n \to 0$, we average Z_c^n over the disorder obtaining the replicated free energy involving only the annealed fields, but generating a quartic coupling between different replicas.

$$< Z_c^n >_{\delta c} = \int \mathcal{D}\vec{r}_a \int \mathcal{D}\chi_{a\alpha\beta} \exp\left\{-\frac{1}{T}F[\vec{r}_a, \chi_{a\alpha\beta}]\right\} \ , \tag{2.5a}$$

$$F[\vec{r}_a, \chi_{a\alpha\beta}] = \sum_{a=1}^{n} F_0[\vec{r}_a, \chi_{a\alpha\beta}] - \frac{d^2\hat{\sigma}}{8T} \sum_{a \neq b} \int d^D x (\partial_\alpha \vec{r}_a \ \ \partial_\alpha \vec{r}_a)(\partial_\beta \vec{r}_b \cdot \partial_\beta \vec{r}_b) \tag{2.5b}$$

In equation (2.5b) F_0 is the replicated free energy for the pure system, $\delta c = 0$. In the above we defined an effective disorder parameter $\hat{\sigma} = (2\mu + D\lambda)^2 \sigma$ and redefined $\lambda \to \lambda - d\hat{\sigma}/T$ in order to eliminate the replica-diagonal terms from the disorder term.

We propose that the crumpled spin-glass phase is characterized by the order parameter $\partial_\alpha r_{ai} \partial_\beta r_{bj}$, a composite quadratic operator of tangent fields from different replicas. The crumpled spin-glass will be characterized by the vanishing of the average tangent field, $< t_{a\alpha i} >= < \partial_\alpha r_{ai} >= 0$ and a nonvanishing average of the spin-glass order parameter, $< t_{a\alpha i} t_{b\beta j} >\neq 0$.

To construct the free energy, which is a function of this spin-glass order parameter, we add to the free energy equation (2.5b) a coupling $d h_{ab\alpha\beta ij} \partial_\alpha r_{ai} \partial_\beta r_{bj}$ to an external field $h_{ab\alpha\beta ij}$. In the standard procedure one constructs the effective free energy by integrating out all of the degrees of freedom and performs a Legendre transform with respect to the external field.[31] However, here we will bypass this procedure, and will utilize a more convenient method similar to the one used to derive the ϕ^4 effective free energy from the lattice Ising model.

Ignoring the unimportant constant term we complete the square between the external field term and the quartic disorder term, obtaining

$$F[\vec{r}_a, \chi_{a\alpha\beta}] = \sum_{a=1}^{n} F_0[\vec{r}_a, \chi_{a\alpha\beta}] - \frac{d^2\hat{\sigma}}{8T} \sum_{a \neq b} \int d^D x \left(\partial_\alpha r_{ai} \partial_\beta r_{bj} + (\frac{4T}{d\hat{\sigma}}) h_{ab\alpha\beta ij} \right)^2 \tag{2.6}$$

We now perform a Hubbard-Stratanovich transformation thereby introducing another set of fields, $Q_{ab\alpha\beta ij}$, [15] and find

$$< Z^n[h_{ab\alpha\beta ij}] >_{\delta c} = \int \mathcal{D}\vec{r}_a \int \mathcal{D}\chi_{a\alpha\beta} \int \mathcal{D}Q_{ab\alpha\beta ij} \exp\left\{-\frac{1}{T}F[\vec{r}_a, \chi_{a\alpha\beta}, Q_{ab\alpha\beta ij}]\right\} \quad (2.7)$$

where

$$F[\vec{r}_a, \chi_{a\alpha\beta}, Q_{ab\alpha\beta ij}] = \sum_{a=1}^{n} F_0[\vec{r}_a, \chi_{a\alpha\beta}]$$
$$+\sum_{a\neq b}\int \mathrm{d}^D x \left[\frac{\hat{\sigma}}{8T}(Q_{ab\alpha\beta ij})^2 - \frac{d\hat{\sigma}}{4T}Q_{ab\alpha\beta ij}\partial_\alpha r_{ai}\partial_\beta r_{bj} - Q_{ab\alpha\beta ij}h_{ab\alpha\beta ij}\right] \quad (2.8)$$

We observe that $Q_{ab\alpha\beta ij}$ couples linearly to the external field. This leads to its physical interpretation as the crumpled spin-glass order parameter.

We are now in the position to construct the effective free energy, by integrating out the fluctuations in the $\vec{r}_a$ degrees of freedom which now appear only quadratically. For now we will work in vanishing external fields. In order to describe the crumpled, the flat and the crumpled spin-glass phases all within the same analysis, we will allow for the possibility of a nonvanishing tangent order parameter, given by $\partial_\alpha \vec{r}_a^{\,o}$. We expand the free energy in equation (2.8) in terms of the fluctuations $\delta\vec{r}_a$ about this ground state, $\vec{r}_a = \vec{r}_a^{\,o} + \delta\vec{r}_a$ and integrate them out. We obtain the expression for the free energy that is correct to the leading order in $1/d$

$$nF_{\rm eff}[\vec{r}_a^{\,o}, \chi_{a\alpha\beta}, Q_{ab\alpha\beta ij}] =$$
$$\sum_{a=1}^{n}\int \mathrm{d}^D x\left[\frac{d}{2}\,\kappa|\nabla^2\vec{r}_a^{\,o}|^2 + \frac{d}{2}\,\chi_{a\alpha\beta}\,(\partial_\alpha\vec{r}_a^{\,o}\;\;\partial_\beta\vec{r}_a^{\,o} - \delta_{\alpha\beta}) - \alpha\frac{d}{2}(\chi_{a\alpha\beta})^2 - \beta\frac{d}{2}(\chi_{a\alpha\alpha})^2\right]$$
$$+\sum_{a\neq b}\int \mathrm{d}^D x\left[\frac{\hat{\sigma}}{8T}(Q_{ab\alpha\beta ij})^2 - \frac{d\hat{\sigma}}{4T}Q_{ab\alpha\beta ij}\partial_\alpha r^o_{ai}\partial_\beta r^o_{bj} + \frac{T}{2}\mathrm{Tr}\ln\{M_{abij}\}\right] , \quad (2.9a)$$

$$M_{abij} = \delta_{ab}\delta_{ij}(\kappa\Delta^2 - \partial_\alpha\chi_{a\alpha\beta}\partial_\beta) + \frac{\hat{\sigma}}{2T}\partial_\alpha Q_{ab\alpha\beta ij}\partial_\beta \quad (2.9b)$$

Note that all the linear terms in $\vec{r}_a^{\,o}$ have vanished by the definition of $\vec{r}_a^{\,o}$ being the minimum of the effective free energy. The matrix M_{abij} is $d\times d$ and hence the last term in the equation (2.9a) coming from the fluctuations in the field $\vec{r}_a$ is proportional to d, as are the tree level terms of the free energy.

Diagramatically, the $\delta\vec{r}_a$ fluctuation contribution to the free energy comes from the sum of all the one-loop graphs constructed from the $\delta\vec{r}_a$ propagators.(see Fig. 1) In this language the d dependence of the free energy is easy to understand. The propagators are proportional to $1/d$ while all the vertices are proportional to d and since there is an equal number of propagators and vertices this gives no net powers of d. However there are $d - D \approx d$ transverse fields that contribute to the loop, leading to the contribution to the free energy that in $d \to \infty$ is proportional to d.

In the pure case $\hat{\sigma} = 0$ one can construct a systematic $1/d$ expansion by also expanding the field $\chi_{a\alpha\beta}$ around the minimum of the free energy as we did with $\vec{r}_a$ and integrating out the

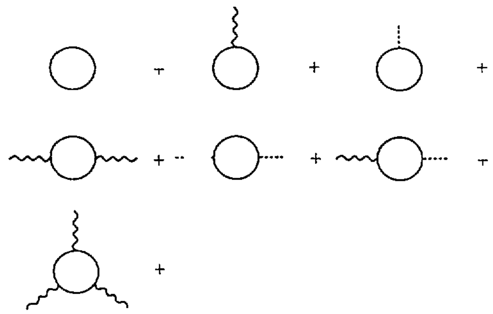

Fig. 1. — One-loop diagrams contributing to the effective free energy. All the terms are proportional to d.

fluctuations. It is easy to see that since the free energy has an overall power of d, the loop expansion of the effective free energy will correspond to this $1/d$ expansion with the sum of n-loop diagrams corresponding to a $(1/d)^n$ term [29-30].

However for $\hat{\sigma} \neq 0$, because of the tensorial structure of the field $Q_{ab\alpha\beta ij}$, the fluctuations in this field may lead to corrections to the effective free energy which are of the same leading order in $1/d$ as the tree level graphs and the $\delta\vec{r}_a$-graphs in figure 1. Summing up these graphs for the $Q_{ab\alpha\beta ij}$ fluctuations is equivalent to a calculation of an effective free energy for a matrix model, which is a well known unsolved problem [32,33].

To compute the effective free energy we use the saddle point method in the equation (2.9a) and take $\chi_{a\alpha\beta}$ and $Q_{ab\alpha\beta ij}$ at their saddle point values. For $\chi_{a\alpha\beta}$ this is exact to the leading order in $1/d$. However for $Q_{ab\alpha\beta ij}$ this approximation is equivalent to considering the spin-glass sector of the free energy in mean-field approximation. It is an improved mean-field theory since the tangent fluctuations are treated exactly in large d and the theory is exact for $\sigma \to 0$.

3. Saddle point equations in $d \to \infty$ limit.

The values of the order parameters $\partial_\alpha \vec{r}_a^{\circ}$ and $Q^{\circ}_{ab\alpha\beta ij}$ are determined by the extremum of the F_{eff}, together with the equation of constraint relating $\chi_{a\alpha\beta}$ to these order parameters.

Assuming that the replica symmetry breaking does not occur until higher order in $1/d$, as it happens in the random anisotropy axis model, [34-35] we look for the saddle point replica symmetric solution of the following form,

$$\vec{r}_a^{\circ} = \zeta x^{\alpha} \hat{e}_{\alpha} \ , \tag{3.1a}$$

$$\chi^{\circ}_{a\alpha\beta} = \chi \delta_{\alpha\beta} \ , \tag{3.1b}$$

$$Q^{\circ}_{ab\alpha\beta ij} = q\delta_{\alpha\beta}\delta_{ij}(1-\delta_{ab}) \tag{3.1c}$$

This ansatz leads to an integral expression for F_{eff},

$$F_{\text{eff}}(\zeta,\chi,q)/L^D = \frac{d}{n}\left[\frac{1}{2}nD\chi(\zeta^2-1) - \frac{nD}{2}\,\chi^2(\alpha+\beta D) + n(n-1)\frac{D}{8T}(\hat{\sigma}q^2 - 2\hat{\sigma}q\zeta^2) + \frac{T}{2}\int_0^{\Lambda}\frac{\mathrm{d}^D k}{(2\pi)^D}\left\{\ln[\kappa k^4 + \chi k^2 - (n-1)\frac{\hat{\sigma}}{2T}qk^2] + (n-1)\ln[\kappa k^4+\chi k^2+\frac{\hat{\sigma}}{2T}qk^2]\right\}\right] . \tag{3.2}$$

In the above equation we introduced a large momentum cutoff Λ which is inversely proportional to the underlying lattice spacing of the membrane. We now take the limit $n \to 0$ as required by the identity in equation (2.4), obtaining the final expression for the effective free energy of the membrane in the limit $d \to \infty$. The extrema of $F_{\rm eff}$ lead to saddle point equations which determines the value of these order parameters,

$$F_{\rm eff}(\zeta,\chi,q)/L^D = d\left[\frac{D}{2}\chi(\zeta^2-1) - \frac{D}{2}\,\chi^2(\alpha+\beta D) - \frac{D\hat{\sigma}}{8T}(q^2-2q\zeta^2) + \frac{T}{4}\int_0^\Lambda \frac{\mathrm{d}^D k}{(2\pi)^D}\left\{2\ln[\kappa k^2+\chi+\frac{\hat{\sigma}}{2T}q] - \frac{q\hat{\sigma}/T}{\kappa k^2+\chi+\frac{\hat{\sigma}}{2T}q}\right\}\right] \qquad (3.3)$$

We calculate the saddle point equations, with ζ, χ and q as variational parameters, by setting the first derivatives of the $F_{\rm eff}$ with respect to these variables to zero.

$$(1-\zeta^2)+2\chi(\alpha+\beta D) = \frac{T}{2D}\int_0^\Lambda \frac{\mathrm{d}^D k}{(2\pi)^D}\left[\frac{2}{\kappa k^2+\chi+\frac{\hat{\sigma}}{2T}q} + \frac{\hat{\sigma}q/T}{(\kappa k^2+\chi+\frac{\hat{\sigma}}{2T}q)^2}\right], \qquad (3.4a)$$

$$\zeta^2 = q\left(1-\frac{\hat{\sigma}}{2D}\int_0^\Lambda \frac{\mathrm{d}^D k}{(2\pi)^D}\frac{1}{(\kappa k^2+\chi+\frac{\hat{\sigma}}{2T}q)^2}\right), \qquad (3.4b)$$

$$\left(\chi+\frac{q\hat{\sigma}}{2T}\right)\zeta = 0\,. \qquad (3.4c)$$

4. Analysis of phase diagram and critical exponents.

Before analyzing the saddle point equations in their full generality we look at the special case of $\hat{\sigma}=0$. This corresponds to a membrane without disorder, and the equations (3.4) reduce to the saddle point equations obtained by Guitter *et al.* in the $d \to \infty$ study of crumpling transition of pure phantom membranes [6] .

$$1-\zeta^2+2\chi(\alpha+D\beta) = \frac{T}{D}\int_0^\Lambda \frac{\mathrm{d}^D k}{(2\pi)^D}\frac{1}{\kappa k^2+\chi}, \qquad (4.1a)$$

$$\chi\zeta = 0\,. \qquad (4.1b)$$

As was found in reference [6] these equations lead to two phases for $D > 2$. The crumpled phase of the membrane is characterized by a vanishing order parameter $\zeta = 0$ and a finite correlation length proportional to $\chi^{-1/2}$ In the flat phase however, the average value of a tangent vector is nonzero and the tangent correlation length diverges. Using equation (4.1b) for $\zeta \neq 0$ inside equation (4.1a) we obtain the temperature dependence of the order parameter ζ in the flat phase,

$$\zeta^2 = \left(1-\frac{T}{T_c}\right). \qquad (4.2)$$

We have defined the crumpling temperature, T_c

$$T_c^{-1} = \frac{1}{D}\int_0^\Lambda \frac{\mathrm{d}^D k}{(2\pi)^D}\frac{1}{\kappa k^2} \qquad (4.3)$$

Equation (4.2) immediately leads to $\beta_\zeta = 1/2$. We also reproduce the exponents $\gamma_\zeta = 2/(D-2)$, $\delta_\zeta = (D+2)/(D-2)$, $\nu_\zeta = 1/(D-2)$ and $\eta_\zeta = 0$ characterizing the flat and crumpled phases for $D < D_\mathrm{u}$. $D_\mathrm{u} = 4$ is the upper critical dimension for the pure membrane above which the membrane is described by classical critical exponents.

We now analyze the full set of saddle point equations for a disordered membrane, equations (3.4). There are three distinct solutions to these equations corresponding to three different possibilities for the values of the pair of order parameters ζ and q.

$$\zeta = 0\ ,\quad q = 0\ , \tag{4.4a}$$

$$\zeta \neq 0\ ,\quad q \neq 0\ , \tag{4.4b}$$

$$\zeta = 0\ ,\quad q \neq 0\ . \tag{4.4c}$$

We identify these three solutions in equations (4.4) with the crumpled phase, flat phase and crumpled spin-glass phase of the membrane, respectively.

We first examine the flat phase with $\zeta \neq 0$, $q \neq 0$. Equation (3.4c) then implies that $\chi + q\hat{\sigma}/2T = 0$. Using this fact and equation (3.4b) inside equation (3.4a) we obtain the temperature and disorder dependence of the order parameters ζ and q inside the flat phase.

$$\zeta^2 = A\left(1 - \frac{T}{T_\mathrm{c}}\right)\left(1 - \frac{\sigma}{\sigma_\mathrm{c}}\right)\ , \tag{4.5a}$$

$$q = A\left(1 - \frac{T}{T_\mathrm{c}}\right) \tag{4.5b}$$

where T_c was defined in equation (4.3) and $\hat{\sigma}_\mathrm{c}$ and A are defined by

$$\sigma_\mathrm{c}^{-1} = \frac{1}{2D}\int_0^\Lambda \frac{\mathrm{d}^D k}{(2\pi)^D}\frac{1}{\kappa^2 k^4}\ , \tag{4.6a}$$

$$A = \frac{1}{1 + (\alpha + D\beta)\hat{\sigma}/T} \tag{4.6b}$$

Since physically ζ is required to be real, equations (4.5) also define the boundaries of the flat phase. It is given by a rectangular region defined by $\hat{\sigma} = \hat{\sigma}_\mathrm{c}$, $T = T_\mathrm{c}$ and the $\hat{\sigma}$, T axes (see Fig. 2a). Outside this region ζ vanishes and the membrane undergoes a transition out of the flat phase. For $\hat{\sigma} \leq \hat{\sigma}_\mathrm{c}$ and as $T \to T_\mathrm{c}$ the transition is to the crumpled phase, while for $T \leq T_\mathrm{c}$ and as $\hat{\sigma} \to \hat{\sigma}_\mathrm{c}$ flat phase is unstable to the crumpled spin-glass phase.

We observe from equation (4.3) that $T_\mathrm{c} = 0$ for $D \leq 2$ and therefore identify $D_{l,T} = 2$ as the lower critical dimension for the existence of finite temperature flat phase. Similarly, equation (4.6a) leads to $\hat{\sigma}_\mathrm{c} = 0$ for $D \leq 4$ giving $D_{l,\sigma} = 4$ as the lower critical dimension for the existence of the flat phase in disordered membranes for $d = \infty$.

From equations (4.5) we obtain the value of the β exponents, that determine how the order parameters ζ and q vanish at the transitions from the flat phase, $\zeta \sim (T_\mathrm{c} - T)^{\beta_{\zeta,T}}$, $\zeta \sim (\hat{\sigma}_\mathrm{c} - \hat{\sigma})^{\beta_{\zeta,\sigma}}$ and $q \sim (T_\mathrm{c} - T)^{\beta_{q,T}}$, where

$$\beta_{\zeta,T} = \frac{1}{2}\ , \tag{4.7a}$$

$$\beta_{\zeta,\sigma} = \frac{1}{2}\ , \tag{4.7b}$$

$$\beta_{q,T} = 1\ . \tag{4.7c}$$

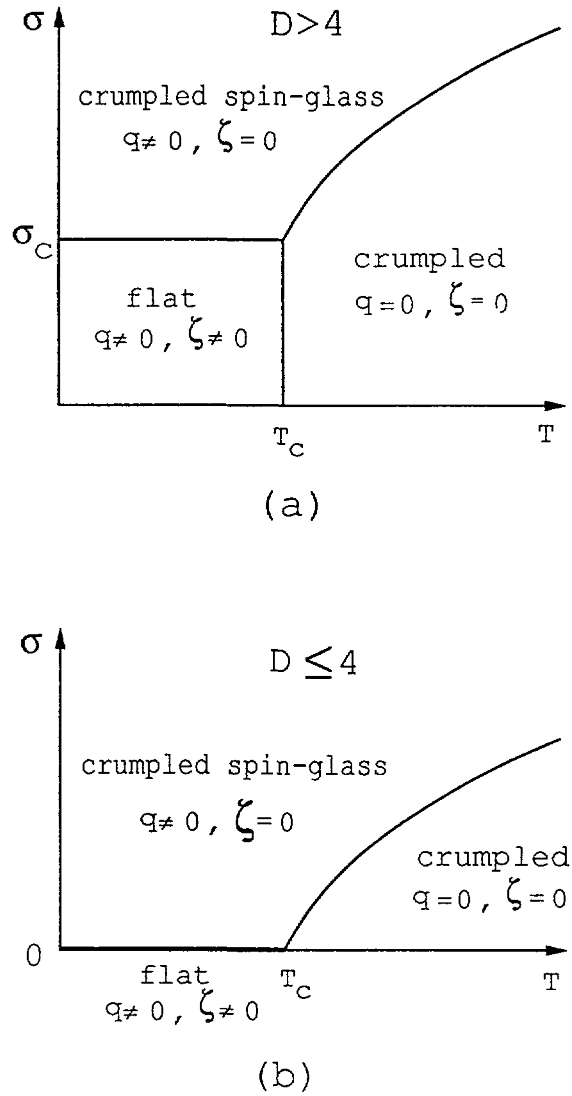

Fig. 2. — Phase diagram for a disordered membranes (a) $D > 4$, (b) $D \leq 4$.

To calculate the spin-glass susceptibility near the transition from the flat phase to the crumpled spin-glass phase we return to the expression for the F_{eff}, equations (2.9) and add an external field $h_{ij\alpha\beta} = h\delta_{ij}\delta_{\alpha\beta}$ conjugate to the spin-glass order parameter. This leads to a modification of only equation (3.4b) which now becomes

$$q\left(1-\frac{\hat{\sigma}}{2D}\int_0^{\Lambda}\frac{\mathrm{d}^D k}{(2\pi)^D}\frac{1}{(\kappa k^2)^2}\right)=h+\zeta^2 \tag{4.8}$$

This equation then immediately leads to the spin-glass susceptibility, $\chi_{\text{sg}} = \partial q/\partial h \sim (\hat{\sigma}_c - \hat{\sigma})^{-\gamma_{\text{sg2}}}$, where

$$\gamma_{\text{sg2}} = 1 \ . \tag{4.9}$$

Now we look inside the spin-glass phase where $q \neq 0$ and $\zeta = 0$. As the flat phase boundary is approached, the tangent susceptibility χ_ζ must diverge since inside the flat phase there is a spontaneous $\zeta \neq 0$. To compute the susceptibility exponent γ_ζ we again turn on an external field f that couples to the tangent order parameter, giving rise to $dD\zeta f$ additional term in the expression for the F_{eff} in equation (2.9). This leads to modification of equation (3.4c),

$$\frac{f}{\zeta} = \chi + \frac{q\hat{\sigma}}{2T} \tag{4.10}$$

Using this equation inside equation (3.4b) we find that $\chi_\zeta = \partial\zeta/\partial f \sim (\hat{\sigma} - \hat{\sigma}_c)^{-\gamma_{\zeta 2}}$ [36]

$$\gamma_{\zeta 2} = \frac{2}{|4-D|}. \tag{4.11}$$

Finally we look at the transition between the spin-glass phase and the crumpled phase. Computing the spin-glass susceptiblity χ_{sg} as before by turning on a small external field h that couples to the the spin-glass order parameter q we find $\chi_{sg} = \partial q/\partial h \sim (\hat{\sigma}_c(T) - \hat{\sigma})^{-\gamma_{sg1}}$, where

$$\gamma_{sg1} = 1 \tag{4.12}$$

The result is identical to equation (4.9) except that $\hat{\sigma}_c$ has been replaced by $\hat{\sigma}_c(T)$ which is nonzero for any D and is defined by

$$\sigma_c^{-1}(T) = \frac{1}{2D}\int_0^\Lambda \frac{\mathrm{d}^D k}{(2\pi)^D}\frac{1}{(\kappa k^2 + \chi(T))^2} \tag{4.13}$$

Equation (4.13) together with equation (3.4a) and $q = 0$ actually also define the phase boundary between the spin-glass and the crumpled phases for $D > 2$, (see Fig. 2)

$$\hat{\sigma}_c(T) - \hat{\sigma}_c \sim (T - T_c)^\phi \ , \tag{4.14}$$

where ϕ is the crossover exponent, [36]

$$\phi = \frac{|D-4|}{D-2} \tag{4.15}$$

5. Estimates of higher order corrections.

In the previous section we have found that $D_{l,\sigma} = 4$ in the limit $d \to \infty$. If this were to carry through for finite d, it would mean that in real physical membranes, $D = 2$, the flat phase can only exist in the absense of disorder, and addition of any amount of impurities will destabilize the flat phase to the crumpled spin-glass phase. However, for finite d, our $d \to \infty$ results do not exclude the existence of a flat phase of size $1/d$. We can see how the $1/d$ corrections can lead to the persistence of the flat phase below $D = 4$ in disordered membranes from the following rough argument [37]. The $1/d$ corrections are known to lead to the anomalous scaling $\kappa(k) \sim k^{-\eta}$, with $\eta = 2/d$ [6,5]. If we use this renormalized result for κ in the expression for $\hat{\sigma}_c$ in equation (4.6a), we find that the lower critical dimension is reduced to $D_{l,\sigma} = 4 - 4/d$, lowering $D_{l,\sigma}$ below 4.

The same conclusion can be reached by applying the Harris criterion to the buckling transition studied in Ref.6. This transition is controlled by the Aronovitz-Lubensky (AL) fixed point, the same fixed point which describes the flat phase of a pure membrane and leads to the anomalous scaling of the elastic moduli described above. [5] Using hyperscaling (below $D = 4$), $\alpha = 2 - D\nu$, with $\nu = 1/(D - 2 + \eta_f)$ we relate the specific heat exponent α to the anomalous dimension of the out-of-plane fluctuation fields. The exact relation between η_f and η_u (the anomalous dimension for the in-plane fluctuation fields) $\eta_u = 4 - D - 2\eta_f$ [38] then leads to $\alpha = -2\eta_u/(D - \eta_u)$. Thus quite generally, using the Harris criterion, the AL fixed point is stable as long as η_u is positive and smaller than D. This is certainly true near $D = 4$ since the ϵ expansion shows that η_u is positive. A similar, although somewhat different argument for the stability of the flat phase at finite temperatures was given in reference [1].

A possible phase diagram for $D \leq 4$ that might result when the $1/d$ corrections are taken into account is illustrated in figure 3. A consistency with the $\epsilon = 4 - D$ expansion of reference [1] (which was performed for arbitrary d) requires the existence of a finite region of flat phase, stable to introduction of uncorrelated disorder. They find that for $D > 4$, at low temperatures, the flat phase is stable to the addition of small amount of disorder. For $D > 4$ our large d results are in agreement with the ϵ expansion. In particular, we find that the scaling of the threshold $\hat{\sigma}_c$ with d is the same as in reference [1] and is another consistency check on our approximation discussed in section 2. Also, for $D \leq 4$ it was found that at $T = 0$ the flat phase is unstable to any amount of disorder, while at finite temperatures the flat phase is stable to weak disorder. This again is consistent with the results of calculations presented here.

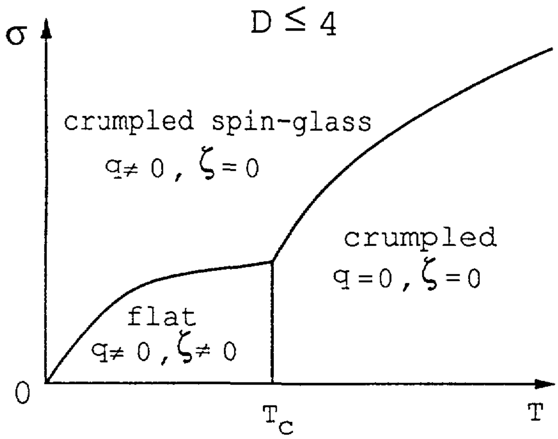

Fig. 3. — Possible phase diagram for a disordered membrane $D \leq 4$ when $1/d$ corrections are taken into account. There is a region of flat phase of size $1/d$.

The crumpled spin-glass phase discussed in this paper deserves further investigation. An open important question is the lower critical dimension D_l^{sg} for this phase. Our saddle point equations, equations (3.4) lead to the glass phase which persists for any D. This we believe is an artifact of our approximation of ignoring the fluctuations in the spin-glass order parameter $Q_{ab\alpha\beta ij}$. If these are properly taken into account they will lead to a finite D_l^{sg}. In view of the results for spin models it appears likely that $D_l^{sg} > 2$ and thus a true phase transition in $D = 2$ phantom tethered membranes might not exist.

Notice however that the flat phase of a pure membrane has a lower D_l^{flat} than naively expected from the analogy with conventional spin models. These differences arise from the long range interaction between the local Gaussian curvatures mediated by the in-plane fluctuations [1] . One cannot exclude that a similar reduction of the lower critical dimension might also occur for the glass phase of the membrane. Even if there is not a true transition most of the characteristics of a glass phase, such as slow dynamics, should still be observed. If this phase exists, an observable quantity is the radius of gyration $R_g = \left(\overline{\langle(\vec{r}(L) - \vec{r}(0))^2\rangle}\right)^{1/2}$, and we expects a nontrivial scaling $R_g \sim L^{\nu_D}$ If we suppose that the bending rigidity is irrelevant in the crumpled glass phase, as might be natural since the phase is crumpled, then we can make a simple Flory (dimensional) argument by balancing the remaining terms in equation (2.1), $\partial_\alpha\vec{r} \cdot \partial_\beta\vec{r} \sim \delta c \delta_{\alpha\beta}$. For uncorrelated disorder considered here, $\delta c \sim L^{-D/2}$ and therefore $\nu_D = (4 - D)/4$, which gives $\nu_D = 1/2$ for $D = 2$. This is to be compared with the scaling $R_g \sim \sqrt{\ln L}$ in the crumpled phase. This difference means that in the glass phase the surface will be larger than in the crumpled phase. Note that similar Flory arguments were used recently for polymers in disordered media and found to give reasonable approximations [39,40].

These type of Flory arguments can be extended to the discussion of a self-avoiding membrane. The self-avoiding interaction is usually taken to be $v_0 \int d^D x_1 \int d^D x_2 \delta^d(\vec{r}(x_1) - \vec{r}(x_2))$ and scales as $L^{2D} R_g^{-d}$ If we assume that R_g scales with exponent $(4 - D)/4$, the phantom membrane result obtained above, then we can compare the scaling of the self-avoidance term, the disorder and elastic energy terms. We find that the self-avoiding interaction is irrelevant for $d > 8D/(4 - D)$, and for a physical $D = 2$ membrane $d > 8$. When the embedding dimension is $d < 8$ the self-avoidance becomes relevant and will affect the scaling of R_g with L. However, the crumpled glass, because it is an energy dominated state, is likely to be more robust to self-avoidance than the usual crumpled phase of tethered membranes.

Finally we observe that the crumpled glass phase can be destroyed by applying an external tension to the membrane's boundaries. The metastable degenerate ground states would disappear and the average of the local tangent would no longer vanish. In this respect an external stress would be analogous to an external magnetic field in spin systems. As the stress is reduced the membrane would slowly return to the glassy phase but with some hysterisis. The line separating the regions of stable and metastable degenerate states is then the analogue of the d'Almeida-Thouless line studied in great detail for the real spin-glasses [41,14].

6. Conclusions.

In this paper we investigated the nature of the disorder activated instability of a flat phase of a random surface, motivated by the previous results of ϵ expansion study of the flat phase. At low temperatures, this phase was found to be unstable to quenched disorder described in terms of random local modification of a ground state metric. The nature of the strong disorder regime was however unclear.

Within the approximations of our model and in the limit $d \to \infty$ we found that the instability is in fact to a crumpled spin-glass phase, characterized by Edwards-Anderson type of order parameter. We have computed the free energy as a function of this order parameter and the tangent order parameter and derived the critical exponents characterizing these phases and the transitions. The Flory type of arguments were then used to estimate the wandering exponent and to discuss the relevance of the self-avoidance in the crumpled phase.

Our analysis suggests that a disordered phantom membrane can undergo transitions between crumpled, flat and crumpled spin-glass phases. At low temperature the transition is to a glassy crumpled phase with a vanishing average tangent order parameter but with a nonvanishing spin-glass order parameter. In this phase the membrane is crumpled but there are finite correlations between crumpled configurations for the different realizations of disorder. The phase diagrams that we obtain are consistent with the ϵ expansion and to a leading order in $1/d$ are very similar to the ones found for the random axis model. However, in the spin systems it was previously found that the ferromagnetic phase is absent below $D = 4$ even if $1/d$ corrections are taken into account [22, 23, 37]. In other words the analogue of the η_f exponent is always zero. In this respect our polymerized membranes are very different from disordered spin systems, with these differences arising due to the presence of nonlinear interactions between the in-plane and out-of-plane phonon degrees of freedom which lead to long-range interactions between the normals to the surface.

Much work remains. Our analysis should be extended to other types of disorder. The disclinations and dislocations can in principle be treated with this approach. A random quenched fluctuations in the extrinsic curvature is another type of disorder that can appear, and might be more relevant to the experiments of reference [20]. This type of disorder is actually more analogous to a random field problem, by contrast with our problem which is rotationally symmetric and hence is closer related to the random exchange problem. The extrinsic curvature disorder was recently studied by Bensimon *et al.* [42] and by Morse and Lubensky [43] within the ϵ expansion. Morse and Lubensky also find a low temperature instability of a flat phase towards a new low temperature flat phase described by different exponents. The strong disorder regime however, also lies outside the range of validity of their expansion. It is possible that a large d expansion could elucidate the character of the disordered phase.

It is also possible that there exists an intermediate *flat* spin-glass phase for disordered membranes, although the method of the present paper does not allow to distinguish between equilibrium flat and *flat* glass phases. It is likely that there is a smooth crossover from the pure flat phase to the disordered flat membrane. In this case the crossover line would be the analog of the Gabay-Toulouse line in the vector spin-glasses [44,14]. One might be able to study the existence of this phase by focussing directly on the flat phase of the membrane and perturbing with small disorder.

We have constructed rough arguments based on previous calculations about the effect of the $1/d$ corrections to our leading order results. However it is important to actually compute these corrections. It might turn out that once these corrections are taken into account it will become necessary to break the replica symmetry with Parisi type of ansatz, [45] as was done for spin systems in the random-axis model [35, 34].

Finally we note that recently it was shown that the spin-glass phase exhibited by the spherical model limit of random anisotropy magnets is a pathology of this limit [46]. Presently it is not clear whether similar pathologies arise in the crumpled glass phase of tethered membranes, and if they do it is important to understand how they affect the results of this work.

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Note added in proof:

After this work was completed we received a long version of the work [43] by Morse and Lubensky where the authors calculate, using a $1/d$ expansion, the exponents of the new $T = 0$ flat phase for random curvature disorder (not considered here). This calculation confirms their ϵ expansion result [43]. Since these authors are not interested in the same region of the phase diagram the rescaling of the parameters with d is different than the one considered here.

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