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Lattice artefacts in SU(3) lattice gauge theory with a mixed fundamental and adjoint plaquette action

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We investigated SU(3) lattice gauge theory with a fundamental and adjoint plaquette term in the action. The purpose is to test whether the choice of a negative adjoint coupling can reduce lattice artefacts and improve the scaling behaviour. To this end, we have studied the finite temperature phase transition, the static potential and the mass of the 0^{++} glueball. We found that indeed the lattice artefacts in e.g. $m_{0^{++}}/T_c$ can be reduced considerably compared with the pure Wilson (fundamental) gauge action at the same lattice spacing.

1. INTRODUCTION

The choice of the gauge SU(3) action plays an important role in QCD lattice simulations, and many efforts have been spent in searching for a formulation where discretisation errors are reduced and/or topological dislocations are suppressed. In this work we investigate a gauge action containing plaquettes in both fundamental and adjoint representation. The motivation for this choice is that in the (β_f, β_a) plane, the pure SU(3) gauge theory in four dimensions has a line of first order phase transitions with an endpoint at [1]

$$(\beta_f, \beta_a) = (4.00(7), 2.06(8)) \quad . \tag{1}$$

The presence of this transition line causes large lattice artefacts; in particular, the mass $m_{0^{++}}$ of the lightest glueball 0^{++} is expected to go to zero as the end-point is approached [2]. As a relic of this behaviour, the estimate of $m_{0^{++}}$ in some physical unit at $\beta_a = 0$, for $5.5 < \beta_f < 6.0$ is much smaller than the continuum result. The purpose of this work is to study whether the discretisation errors can be reduced by choosing a negative value of β_a, i.e. by moving away from the transition line.

2. THE ACTION

We consider lattice SU(N) gauge theory with a mixed fundamental-adjoint action,

$$S = \beta_f \sum_P \left[1 - \frac{1}{N}\mathrm{ReTr}_f U_P\right] + \beta_a \sum_P \left[1 - \frac{1}{N^2}\mathrm{Tr}_f U_P^\dagger \mathrm{Tr}_f U_P\right] \quad , \tag{2}$$

where β_f, β_a are the fundamental and adjoint couplings and U_P is the elementary plaquette. We adopted 4-dimensional lattices with spatial extensions aN_s and temporal extension aN_t, with periodic boundary conditions in all directions. The Yang-Mills theory is recovered in the naive continuum limit if the bare coupling g_0 is defined by $6/g_0^2 = \beta_f + 2\beta_a$.

Details on the simulation algorithm and its performance are given in [3].

3. THE FINITE TEMPERATURE PHASE TRANSITION

For a lattice with N_t points in the time direction, in the limit $N_s \to \infty$, the deconfinement temperature is given by

$$\frac{1}{T_c} = N_t a(\{\beta_f, \beta_a\}_c) \quad , \tag{3}$$

where $\{\beta_f, \beta_a\}_c$ indicates the critical coupling. In our study we kept fixed the adjoint coupling to

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Table 1
Numerical results for the finite temperature phase transition obtained for the fundamental-adjoint gauge action.

N_t; β_a	0.0	–2.0	–4.0
2	5.0948(6)	6.4475(6)	7.8477(6)
3	5.5420(3)	7.1603(3)	8.8357(4)
4	5.6926(2)	7.4433(3)	9.2552(6)
6	-	7.8056(5)	9.7748(11)

$\beta_a = 0, -2.0, -4.0$ and determined $\beta_{f,c}$ for $N_t = 2, 3, 4, 6$ by adopting the method discussed in [4].

Our results for $\beta_{f,c}$ are summarised in Table 1; for $\beta_a = 0$ we find a substantial agreement with other results present in the literature [5, 6, 7, 8]. For $\beta_a = 0$, $N_t = 6$ we performed no own simulation, but used in the following the result $\beta_{f,c} = 5.89405(51)$ of ref. [6].

4. THE STATIC POTENTIAL

In order to fix the scale we computed the static potential (at zero temperature) and extracted the string tension $a^2\sigma$. The static potential has been computed through the Polyakov loop correlation function:

$$aV(r) = -\frac{1}{N_t}\left[\log\langle P(x)^* P(y)\rangle + \epsilon\right] \quad , \tag{4}$$

where $y = x + r\hat{1}$ and ϵ is the correction due to excited states in the string spectrum. Here we used a large temporal extension $aN_t >> 1/T_c$ to ensure that finite-N_t corrections are negligible; in particular we adopted $N_t = 6/(aT_c)$ for all computations. The string tension has then been evaluated from the ansatz

$$V(r) = \sigma r + \mu - \frac{\pi}{12r}\left(1 + \frac{b}{r}\right), \tag{5}$$

where b has been recently shown from theoretical principles to be zero [9].[2] Moreover, we adopted the tree-level improved distance r_I, defined such that the force at tree-level has no lattice artefacts. In order to compute the static potential up to large distances by keeping the statistical uncertainties under control, we adopted a variant of the algorithm proposed by Lüscher and Weisz [10]. The main difference consists in using factorisation in the spatial directions, in addition to the one in the temporal direction. The full details of the procedure are given in [3]. For $1/(aT_c) = 4$ we were able to extract $a^2\sigma$ from the force at $r/a = 7$ ($\beta_a = 0$); in the other cases our final values were taken from $r/a = 4$ and $r/a = 6$. In all computations we where confident the quoted error also covers possible systematic uncertainties. The numerical results for $a^2\sigma$ are reported in [3].

The dimensionless quantity $T_c/\sqrt{\sigma}$ as function of the lattice spacing is plotted in fig. 1, together with other values obtained for the Wilson action [8]. We notice that for $\beta_a = -2$ and -4 the estimate for $T_c/\sqrt{\sigma}$ is closer to the continuum limit than for $\beta_a = 0$. However, the difference between $1/(aT_c) = 3$ and $1/(aT_c) = 4$ is larger than that for the different values of β_a at fixed $1/(aT_c)$.

5. THE 0^{++} GLUEBALL MASS

The mass of the lightest glueball is expected to be most sensitive to the choice of the action in the (β_f, β_a) plane. The 0^{++} glueball mass has been computed through the connected correlation function between spatial Wilson loops, by adopting substantially the same method used in [11]. Also for the glueball 2-point function we made use of an error reduction procedure based on the idea proposed in [10], and already applied for the computation of glueball masses [12]. We extracted the glueball masses at $t/a = 2, 3$ for $1/(aT_c) = 2, 3$ and $t/a = 3, 4$ for $1/(aT_c) = 4, 6$. Fig. 2 shows our final results for $m_{0^{++}}/T_c$ as function of $(aT_c)^2$; here the errors are dominated by the uncertainties on $m_{0^{++}}$. By averaging several results in the literature [13, 14, 15, 16] for $m_{0^{++}} r_0$ and then using the continuum limit relation [11]

$$T_c r_0 = 0.7498(50), \tag{6}$$

we obtain

$$m_{0^{++}}/T_c|_{a=0} = 5.73(9) \tag{7}$$

as estimation of the continuum result. At $a \simeq 0.11$fm we do observe a moderate reduction of the lattice artefacts by using $\beta_a < 0$ with respect to

[2] In our evaluation we considered $b = 0.04$fm, which was evaluated numerically in [10]. Nevertheless the effect of having $b \neq 0$ is only minor and not crucial for our computation of σ.

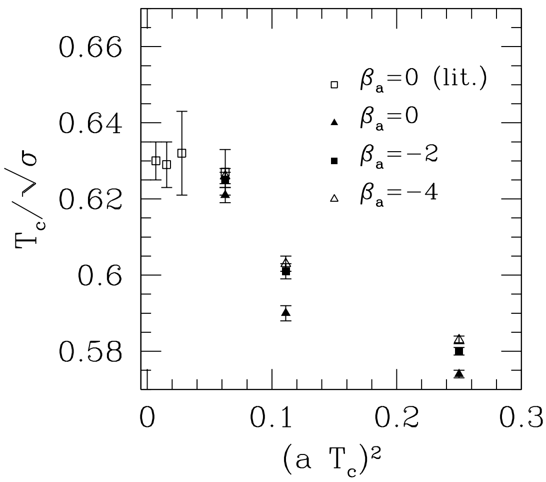

Figure 1. *Results for $T_c/\sqrt{\sigma}$ as a function of $(aT_c)^2$. In addition, we report results from [8] for the Wilson action at smaller lattice spacings.*

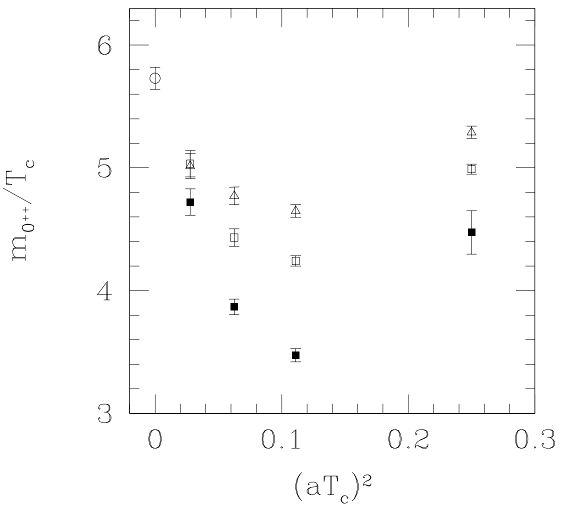

Figure 2. *$m_{0^{++}}/T_c$ for $\beta_a = 0$ (filled squares) $\beta_a = -2$ (open squares) and $\beta_a = -4$ (triangles) as function of $(aT_c)^2$. The circle gives the continuum results extracted from the literature.*

the usual Wilson action ($\beta_a = 0$). For $\beta_a = 0$, the deviation from the continuum result of eq. (7) amounts to $\sim$ 18%, while for $\beta_a = -2, -4$ it slightly decreases to $\sim$ 12%.
At $a \simeq 0.17$fm one observes discretisation errors of $\sim$ 40% for the Wilson action, while for the mixed action they amount to $\sim$ 25% for $\beta_a = -2$ and $\sim$ 20% for $\beta_a = -4$.

6. CONCLUSIONS

We investigated scaling properties of a SU(3) lattice gauge action with plaquette terms in the fundamental and in the adjoint representation, with negative adjoint coupling β_a. By studying the scaling behaviour of the quantity $T_c/\sqrt{\sigma}$ for the different β_a at our disposal, we did not observe a significant improvement at negative adjoint couplings in comparison to the Wilson case $\beta_a = 0$. The values obtained with negative β_a are a little closer to the continuum limit. As expected, the mass $m_{0^{++}}$ of the lightest glueball is more sensitive to the variation of β_a. We investigated the scaling behaviour of the dimensionless quantity $m_{0^{++}}/T_c$. Here indeed, we observed a significant reduction of the lattice artefacts for negative β_a. At $a \simeq 0.17$ fm, the lattice artefacts for $\beta_a = 0$ are 40%, while for $\beta_a = -4$ they decrease to 20%.

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